Wed wk 2

Today: finish 4-5 & 4-6

4-4: L'Hôpital's Rule:

If $\lim_{x\to c} \frac{f(x)}{g(x)} = \begin{cases} \frac{0}{0} \\ \frac{\infty}{\infty} \end{cases}$ then replace $\frac{f(x)}{g(x)}$ w/ $\frac{f'(x)}{g'(x)}$

$$= \lim_{x\to c} \frac{f'(x)}{g'(x)}$$

Select all the indeterminate forms.

- [x] $\frac{0}{0}$
- [x] $\infty - \infty$
- [ ] $\frac{1}{0}$ $= \infty$
- [ ] $\infty \cdot \infty$ $= \infty$
- [ ] $\frac{0}{\infty}$ $= 0$
- [x] 1^∞
- [ ] 0^1 $= 0$
- [x] $0 \cdot \infty$ $\to 0$, $\to \infty$

$\boxed{\frac{0}{0}}$

① $\lim_{x\to 0} \frac{3x^2+x}{x^2+5x} \overset{D.S.}{=} \frac{0}{0}$

L'H

$\lim_{x\to 0} \frac{6x+1}{2x+5} \overset{D.S.}{=} \frac{1}{5}$

② $\lim_{x\to 0} \frac{5x}{x^2+100x} = \frac{0}{0}$

L'H

$\lim_{x\to 0} \frac{5}{2x+100} = \frac{5}{100} = \frac{1}{20}$

③ $\lim_{x\to\infty} \frac{3x^2+x}{x^2+5x} \overset{D.S.}{=} \frac{3(\infty)^2+\infty}{\infty^2+5(\infty)} = \frac{\infty}{\infty}$

L'H

$\lim_{x\to\infty} \frac{6x+1}{2x+5} = \frac{\infty}{\infty}$

L'H

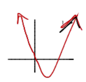
$\lim_{x\to\infty} \frac{6}{2} = 3$

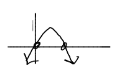
④ $\lim_{x\to\infty} \frac{5x}{x^2+100x} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{5}{2x+100} = \frac{5}{\infty} = 0$

$\boxed{\infty - \infty}$

$\lim_{x\to\infty} x^2 - x \overset{D.S.}{=} \infty - \infty$ vs $\lim_{x\to\infty} x - x^2 = \infty - \infty$





$\boxed{1^\infty}$

$\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e$

$\overset{D.S.}{=} \left(1+\frac{1}{\infty}\right)^\infty = 1^\infty$

$\boxed{0 \cdot \infty}$

$\lim_{x\to\infty} \frac{1}{x} \cdot e^x \overset{D.S.}{=} \frac{1}{\infty} \cdot e^\infty = 0 \cdot \infty$ L'H doesn't apply

$= \lim_{x\to\infty} \frac{e^x}{x} = \frac{\infty}{\infty}$ L'H! $\Rightarrow \lim_{x\to\infty} \frac{e^x}{1} = e^\infty = \infty$

transform / re-write so L'H applies (move the term going to 0 into denom)

$\lim_{x\to\infty} \frac{1}{x^{10}} \cdot (x^2+10) = 0 \cdot \infty$

$= \lim \frac{x^2+10}{x^{10}} \overset{L'H}{=} 0$

Question 3 of 16

State whether L'Hôpital's Rule applies or not.

$$\lim_{x \to -9} \frac{x^2 - 81}{45 - 4x - x^2} \overset{D.S.}{=} \frac{(-9)^2 - 81}{45 - \underbrace{4(-9)}_{} - \underset{\overset{\parallel}{81}}{(-9)^2}} = \frac{81 - 81}{45 + 36 - 81} = \frac{0}{0}$$

L'H yes

$$= \lim_{x \to -9} \frac{2x}{-2x - 4} = \frac{-18}{18 - 4} = \frac{-18}{14}$$

$$l = \lim_{x \to 4} (5-x)^{\frac{1}{x-4}} \overset{D.S}{=} 1^{\frac{1}{0}} = 1^{\infty} \quad ?$$

Idea: re-write so L'H applies (use ln)

① set $l = \lim_{x \to 4} (5-x)^{\frac{1}{x-4}}$, find $\underline{\underline{l}}$

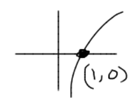
② hit w/ ln : $\ln(l) = \ln\left(\lim_{x \to 4} (5-x)^{\frac{1}{x-4}}\right)$

key: ln and lim $\underline{\underline{\text{commute}}}$ (swap order)
cts

$$= \lim_{x \to 4} \ln(5-x)^{\frac{1}{x-4}}$$

③ use log properties

$$= \lim_{x \to 4} \frac{1}{x-4} \cdot \ln(5-x)$$



?

$$\overset{D.S.}{=} \frac{1}{4-4} \cdot \ln(5-4) = \frac{1}{0} \cdot \ln(1) = \infty \cdot 0$$

④ re-write $= \lim_{x \to 4} \frac{\ln(5-x)}{x-4} \overset{D.S.}{=} \frac{\ln(1)}{0} = \frac{0}{0}$ L'H

⑤ $\ln(l) = \lim_{x \to 4} \frac{\frac{1}{5-x}(-1)}{1} \overset{D.S}{=} \frac{\frac{-1}{5-4}}{1} = \frac{-1}{1} = -1$

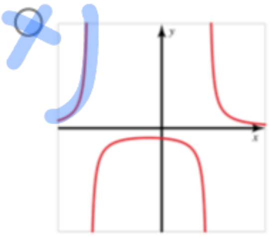
⑥ $e^{\ln(l)} = e^{-1} \Rightarrow l = e^{-1} = \frac{1}{e}$

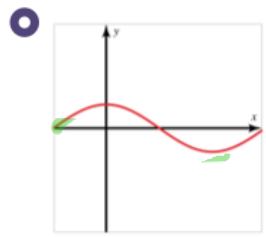
Question 1 of 5

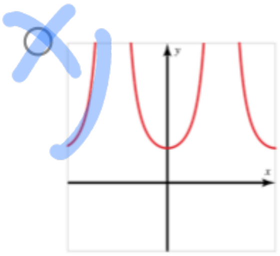
Inc c.u. dec c.u.

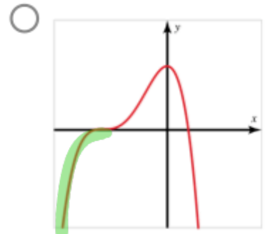
Consider the function f for which f' and f'' have the following sign combinations: $++, -+, ++, -+$. Note that the first sign in each pair represents the sign of the first derivative, and the second sign in each pair represents the sign of the second derivative.

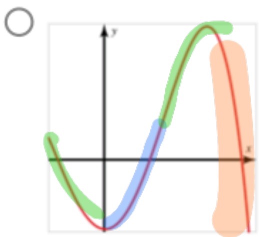
Select the graph of f.











INC / DEC + CU / CD

$f(x) = x^3 - 4x^2 + 8x + 3$

$f'(x) = 3x^2 - 8x + 8 = 0$

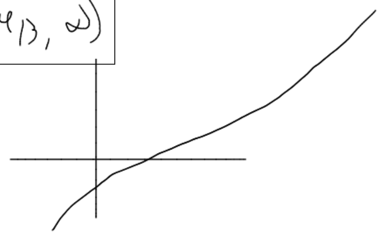
Q.F.

$$x = \frac{8 \pm \sqrt{64 - 4 \cdot 3 \cdot 8}}{2 \cdot 3} = \frac{8 \pm \sqrt{64 - 96}}{6} = \frac{8 \pm \sqrt{-32}}{6} = \frac{8 \pm 4\sqrt{-2}}{6}$$

no real sol's

C.D. $(-\infty, 4/3)$

C.U. $(4/3, \infty)$



$f''(x) = 6x - 8 = 0$

$x = \frac{8}{6} = \frac{4}{3}$

$f''(0) = -8 < 0$

$-$ $+$

$x=0$ $4/3$ $x=2$ $f''(x)$