Wed wk 9

4-5 L'Hopital's Rule

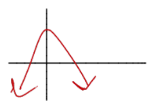
if $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = \begin{cases} \frac{0}{0} \\ \frac{\infty}{\infty} \end{cases}$ then replace $\dfrac{f(x)}{g(x)}$ with $\dfrac{f'(x)}{g'(x)}$

Select all the indeterminate forms.

- [x] $\frac{0}{0}$
- [x] $\infty - \infty$
- [ ] $\frac{1}{0} = \infty$
- [ ] $\infty \cdot \infty = \infty$
- [ ] $\frac{0}{\infty} = 0$
- [x] 1^∞
- [ ] $0^1 = 0$
- [x] $0 \cdot \infty$

$\boxed{\infty - \infty}$

$\lim\limits_{x\to\infty} x^2 - x = +\infty$ vs $\lim\limits_{x\to\infty} x - x^2 = -\infty$



$\infty^2 - \infty$
$=$
$\infty - \infty$

$\boxed{\frac{0}{0}}$

① $\lim\limits_{x\to 0} \dfrac{3x^2 + x}{x^2 + 100x} \overset{D.S}{=} \dfrac{0}{0} = \dfrac{1}{100}$

L'H $\lim\limits_{x\to 0} \dfrac{6x+1}{2x+100} = \dfrac{6\cdot 0+1}{2\cdot 0+100} = \dfrac{1}{100}$

② $\lim\limits_{x\to 0} \dfrac{5x^3 + x}{x^2 + x} \overset{D.S.}{=} \dfrac{0}{0} = 1$

L'H $\lim\limits_{x\to 0} \dfrac{15x^2+1}{2x+1} = \dfrac{1}{1}$

$\boxed{1^\infty}$

$\lim\limits_{x\to\infty} \left(1 + \dfrac{1}{x}\right)^x \overset{D.S.}{=} 1^\infty \neq 1$

Question 3 of 16

State whether L'Hôpital's Rule applies or not.

$$\lim_{x\to-9} \frac{x^2-81}{45-4x-x^2} = \frac{(-9)^2-81}{\underbrace{45-4(-9)}_{+36 \,\to\, 81} - \underbrace{(-9)^2}_{81}} = \frac{0}{0} \quad \text{L'H applies}$$

$$\lim_{x\to-9} \frac{2x}{-4-2x} \overset{D.S.}{=} \frac{-18}{-4-2(-9)} = \frac{-18}{-4+18} = \frac{-18}{14}$$

$$\lim_{x \to \infty} \frac{x}{e^x} =$$

Evaluate the limit using L'Hôpital's Rule.

$$\lim_{x \to 0} \frac{3x^3}{\sin(x) - x} = \lim_{x \to 0} \frac{9x^2}{\cos x - 1} \quad \frac{0}{\cos 0 - 1} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{18x}{-\sin x} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{18}{-\cos x} = \frac{18}{-1} = -18$$

$$\lim_{x \to 4} (5-x)^{\frac{1}{x-4}} = 1^{\frac{1}{0}} = 1^{\infty} \quad \boxed{\text{NO L'H!}}$$

another "ln" algorithm

① set $y = \lim_{x \to 4} (5-x)^{\frac{1}{x-4}}$, goal: find y

② hit w/ ln

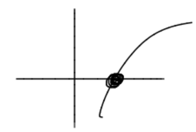
continuous

key: ln and lim commute (swap order)

$$\ln(y) = \ln\left(\lim_{x \to 4} (5-x)^{\frac{1}{x-4}}\right)$$

swap order

$$= \lim_{x \to 4} \left(\ln (5-x)^{\frac{1}{x-4}}\right)$$



log property

$$= \lim_{x \to 4} \frac{1}{x-4} \cdot \ln(5-x) \overset{D.S.}{=} \frac{1}{0} \cdot \overbrace{\ln(5-4)}^{\ln(1)}$$

$\infty \cdot 0$ $\boxed{\text{NO L'H}}$

③ move the thing causing ∞ into denom ...

$$= \lim_{x \to 4} \frac{\ln(5-x)}{x-4} = \frac{\ln(1)}{0} = \frac{0}{0} \quad \text{L'H}$$

④ $\ln(y) \overset{L'H}{=} \lim_{x \to 4} \dfrac{\frac{(-1)}{5-x}}{1} = \dfrac{\frac{-1}{5-4}}{1} = -1$

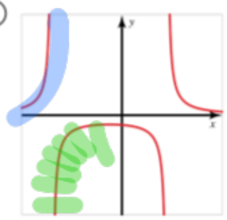
⑤ $\ln(y) = -1 \overset{\text{hit w/ e}}{\rightsquigarrow} e^{\ln(y)} = e^{-1} \Rightarrow y = e^{-1} = \frac{1}{e}$

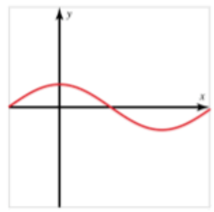
Inc C.U

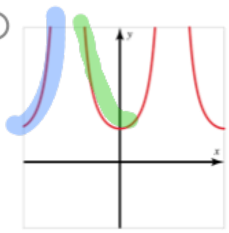
Consider the function f for which f' and f'' have the following sign combinations: $++, -+, ++, -+$. Note that the first sign in each pair represents the sign of the first derivative, and the second sign in each pair represents the sign of the second derivative.

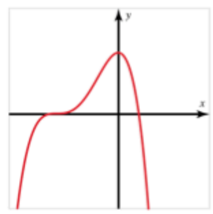
dec C.U.

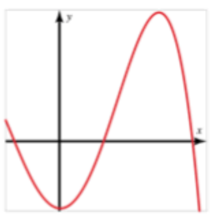
Select the graph of f.











>0 <0 >0 <0

on $f(x) = x^3 - 4x^2 + 8x + 3$ is increasing, decreasing, concave up, and concave down.

f' f''

$f'(x) = 3x^2 - 8x + 8$

$= 0$

Quad Form $x = \dfrac{8 \pm \sqrt{64 - 4 \cdot 3 \cdot 8}}{2 \cdot 3} = \dfrac{8 \pm \sqrt{-12}}{6}$ = no real sols.

$f''(x) = 6x - 8 = 0$ $x = \frac{8}{6} = \frac{4}{3}$ $\Rightarrow$ P.O.I

C.D. $(-\infty, 4/3)$ C.U. $(4/3, \infty)$

$\ominus$ $\oplus$ $f''(x)$

$4/3$