

Integration by parts—guidelines

Example:

$$\int x \ln x \, dx = ??$$

First “select” your u and dv .

Selection guidelines: the derivative of u should be less complex than u , your dv should have a relatively simple antiderivative, *and* you need to “use up” everything in the integral sign. For Math 134—following “LIATE” to pick your u normally works:

- L \Rightarrow logarithmic functions
- I \Rightarrow inverse trigonometric functions
- A \Rightarrow algebraic functions
- T \Rightarrow trigonometric functions
- E \Rightarrow exponential functions

Then find du and v based on your selection.

$$\begin{aligned} u &= \ln x & dv &= x \, dx \\ du &= \frac{1}{x} \, dx & v &= \int x \, dx = \frac{1}{2}x^2 \end{aligned}$$

Now apply the integration by parts rule $\int u \, dv = uv - \int v \, du$ to get

$$\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 * \frac{1}{x} \, dx$$

So

$$\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x \, dx = \frac{1}{2}x^2 \ln x - \left(\frac{1}{2}\right) \left(\frac{x^2}{2}\right) + C = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

Check:

$$\begin{aligned} \frac{d}{dx} \left(\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 \right) &= \frac{d}{dx} \left(\frac{1}{2}x^2 \ln x \right) - \frac{d}{dx} \left(\frac{1}{4}x^2 \right) \\ &= x \ln x + \left(\frac{1}{2}x^2\right) \left(\frac{1}{x}\right) - \frac{1}{4}(2x) = x \ln x + \frac{1}{2}x - \frac{1}{2}x = x \ln x \end{aligned}$$

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