Finding Taylor series the "easy" way . . . .

$$e^x = 1 + x + \frac{1}{2!}\,x^2 + \frac{1}{3!}\,x^3 + \cdots + \frac{1}{n!}\,x^n + \cdots +$$

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots$$

1. Use the above to find the Taylor series at 0 for e^{2x^2}.

2. Use the above to find the Taylor series at 0 for

$$\frac{1}{1-8x^3}.$$

3. Use the series above to find the Taylor series at 0 for

$$f(x) = \frac{x}{1-x^2}.$$

4. Use #3 to find the Taylor series at 0 for

$$g(x) = \frac{1 + x^2}{(1 - x^2)^2}$$

5. Use the following series to find the Maclaurin series for $\cos x$:

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots = \sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1}$$

. Find the intervals of convergence for

#1:

#2:

#3:

#4:

#5: