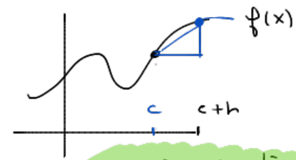
Thursday

1. **Calculus I Review**
    a. **What is Calculus I?**
        i. **Precise way of calculating change (in functions)**
        ii. **Two Branches: Differential & Integral**
            1. **Both depend on the idea of an infinite limit**

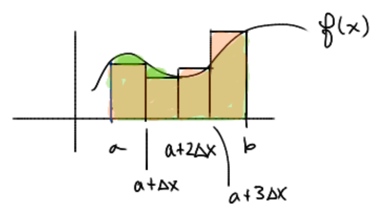


The Derivative

$$\boxed{f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}}$$

Link: https://www.desmos.com/calculator/t6sxoyxyty

The Definite Integral



w/ $\Delta x = \frac{b-a}{n}$

$$\boxed{\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x^*) \cdot \Delta x}$$

Link: https://www.desmos.com/calculator/tgyr42ezjq

Derivatives:

Power Rule:

$f(x) = x^n$, $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$

$= \lim_{h\to 0} \frac{\binom{n}{0}x^n + \binom{n}{1}x^{n-1}h + \binom{n}{2}x^{n-2}h^2 + \dots + \binom{n}{n}x^0}{h} = \frac{\binom{n}{1}x^{n-1}h}{h} = nx^{n-1}$

The pink terms are equal

The white terms vanish as $h \to 0$.

TRIG.

$f(x) = \sin(x)$ $f'(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h} = \lim_{h\to 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}$

$= \lim_{h\to 0} \sin(x) \cdot \left(\frac{\cos(h) - 1}{h}\right) + \cos(x) \lim_{h\to 0}\left(\frac{\sin(h)}{h}\right)$

$= \sin(x) \cdot 0 + \cos(x) \cdot 1 = \cos(x)$

These ideas lead to these formulas:

Functions	Derivatives
x^n	nx^{n-1}
$\ln(x)$	$1/x$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\csc(x)\cot(x)$
$\cot(x)$	$-\csc^2(x)$
e^x	e^x
$\sin^{-1}(x)$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1}(x)$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1}(x)$	$\frac{1}{1+x^2}$
$\sec^{-1}(x)$	$\frac{1}{x\sqrt{x^2-1}}$
$\csc^{-1}(x)$	$\frac{-1}{x\sqrt{x^2-1}}$
$\cot^{-1}(x)$	$\frac{-1}{1+x^2}$

Functions	Anti-Derivatives
x^n	$\frac{x^{n+1}}{n+1}$
$1/x$	$\ln\lvert x\rvert$
$\cos(x)$	$\sin(x)$
$\sin(x)$	$-\cos(x)$
$\sec^2(x)$	$\tan(x)$
$\sec(x)\tan(x)$	$\sec(x)$
$\csc(x)\cot(x)$	$-\csc(x)$
$\csc^2(x)$	$-\cot(x)$
e^x	e^x
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1}(x)$
$\frac{1}{\sqrt{1-x^2}}$	$-\cos^{-1}(x)$
$\frac{1}{1+x^2}$	$\tan^{-1}(x)$
$\frac{1}{x\sqrt{x^2-1}}$	$\sec^{-1}(x)$
$\frac{1}{x\sqrt{x^2-1}}$	$-\csc^{-1}(x)$
$\frac{-1}{1+x^2}$	$\cot^{-1}(x)$

why is $\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$?

① set $y = \sin^{-1}(x)$. Goal, find $y' = \frac{dy}{dx}$.

(chain rule)

② so $\sin(y) = x$ & $\frac{d}{dx}(\sin(y)) = \frac{d}{dx}(x)$ so $\cos(y) \cdot \frac{dy}{dx} = 1$

③ $\frac{dy}{dx} = \frac{1}{\cos(y)}$ & since $\sin^2(y) + \cos^2(y) = 1$ it follows that $\cos(y) = \sqrt{1 - \sin^2(y)}$
w/ $\sin(y) = x$ from step ②

④ $y' = \frac{1}{\sqrt{1-x^2}}$

u- substitution

In this fact $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$ there's nothing special about x. (what really matters is that it's a continuously changing real variable)

So if $u = f(x)$ then $\frac{du}{dx} = f'(x)$ or $du = f'(x)\,dx$. Meaning

$\int (x^2+7)^5 \cdot (2x)\,dx$ can be solved by

$u = x^2+7$, $\frac{du}{dx} = 2x$ or $du = 2x\,dx$ so the integral is transformed into

$$\int u^5\,du = \frac{u^6}{6} + c$$

Another view on

$$\int \cos(x)\,dx = \sin(x)$$

LINK: https://www.desmos.com/calculator/aszy5en1qt