some not-so-basic substitution problems

1. Use $u = x + 2 \Longrightarrow du = dx$ and $u - 2 = x$.

$$\int \frac{x}{\sqrt{x+2}}\,dx = \int \frac{u-2}{\sqrt{u}}\,du = \int (u-2)u^{-1/2}\,du$$
$$= \int u^{1/2} - 2u^{-1/2}\,du = \frac{2}{3}u^{3/2} - 4u^{1/2} + C$$
$$= \frac{2}{3}(x+2)^{3/2} - 4(x+2)^{1/2} + C$$

2. Use $u = x^2 - 1 \Longrightarrow du = 2x\,dx$.

$$\int \frac{x}{\sqrt{x^2-1}}\,dx = \int \frac{1}{2} \cdot \frac{1}{\sqrt{x^2-1}} \cdot 2x\,dx$$
$$= \int \frac{1}{2} \cdot \frac{1}{\sqrt{u}}\,du = \int \frac{1}{2}u^{-1/2}\,du = u^{1/2} + C$$
$$= \sqrt{x^2-1} + C$$

3. Use $u = x^2 - 1 \Longrightarrow du = 2x\,dx$ and $u + 1 = x^2$.

$$\int \frac{x^3}{\sqrt{x^2-1}}\,dx = \int \frac{1}{2} \cdot \frac{x^2}{\sqrt{x^2-1}} \cdot 2x\,dx$$
$$= \int \frac{1}{2} \cdot \frac{u+1}{\sqrt{u}}\,du = \int \frac{1}{2}(u+1)u^{-1/2}\,du$$
$$= \int \frac{1}{2}u^{1/2} + \frac{1}{2}u^{-1/2}\,du = \frac{1}{3}u^{3/2} + u^{1/2} + C$$
$$= \frac{1}{3}(x^2-1)^{3/2} + (x^2-1)^{1/2} + C$$

4.

$$\int \frac{2x+1}{x^2+1}\,dx == \int \frac{2x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx$$
$$= \ln(x^2+1) + \tan^{-1} x + C$$

5. Use $u = \sin x \Longrightarrow du = \cos x\,dx$.

$$\int \sin^3 x \cos x\,dx = \int u^3\,du = \frac{1}{4}u^4 + C = \frac{1}{4}\sin^4 x + C$$

6. use $u = \tan x \Longrightarrow du = \sec^2 x\,dx$.

$$\int \tan x \sec^2 x\,dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\tan^2 x + C$$

7. Use $u = x^3 \Longrightarrow du = 3x^2\,dx$.

$$\int \frac{3x^2}{x^6+1}\,dx = \int \frac{1}{u^2+1}\,du = \tan^{-1} u + C = \tan^{-1}\left(x^3\right) + C$$

8. Use $u = x^2 \Longrightarrow du = 2x\,dx$.

$$\int \frac{2}{x\sqrt{x^4-1}}\,dx = \int \frac{1}{x^2\sqrt{(x^2)^2-1}}(2x)\,dx$$
$$= \int \frac{1}{u\sqrt{u^2-1}}\,du = \sec^{-1} u + C$$
$$= \sec^{-1}\left(x^2\right) + C$$