Friday

1. U-sub basics
2. Not so u-sub basics

Friday: less obvious u-subs

Monday: trig

$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

$\cos^2 x = 1 - \sin^2 x$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\sin^2 x = 1 - \cos^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\frac{\cos 2x + 1}{2} = \cos^2 x$

$\int \cos^2 x \, dx = \int \frac{\cos 2x + 1}{2} dx = \frac{1}{2}\int \cos 2x \, dx + \frac{1}{2}\int dx$

$= \boxed{\frac{1}{4}\sin 2x + \frac{1}{2}x + C}$

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② $\int \sin^3(x)\,dx = \int \sin x \cdot \sin^2 x \, dx = \int \sin x\,(1-\cos^2 x)\,dx = \int \sin x - \int \cos^2 x \sin x \, dx$

③ $\int \cos^3 x \sin^2 x \, dx = \int \cos x\,(1-\sin^2 x)\sin^2 x) = \int \sin^2 x \cos x - \int \sin^4 x \cos x$

④ $\int \cot x = \int \frac{\cos x}{\sin x}$

⑤ $\int \csc x \, dx = \int \csc x \frac{\csc x + \cot x}{\csc x + \cot x} dx = \int \frac{\csc^2 x + \csc x \cot x \, dx}{\csc x + \cot x}$

$u = \csc x + \cot x$

$du = -\csc x \cot x - \csc^2 x$

$= -[\csc^2 x + \csc x \cot x]\,dx$

$= -\int \frac{du}{u} = -\ln|\csc x + \cot x|$

$$\int \sin^2 x \cdot \cos^2 x \, dx = \int (1-\cos^2 x)\cos^2 x = \int \cos^2 x - \cos^4 x \, dx$$

$$= \int \frac{\cos 2x + 1}{2}$$

$$\int \left(\frac{1-\cos(2x)}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx$$

$$\int \frac{1}{4} + \frac{\cos 2x}{4} - \frac{\cos 2x}{4} - \frac{\cos^2(2x)}{4}$$

$$\int \frac{1}{4} - \frac{\cos^2(2x)}{4} \, dx = \frac{1}{4}x - \int \frac{1 + \cos(4x)}{8}$$

$$\int \tan^2 x \, dx = \int \sec^2 x - 1 = \tan x - x + C$$

reduction formula

FRI — WK 1

Substitution

1. change of var. / def. integral

2. basics

① $\int e^{7x+1}\,dx$

② $\int 3x\cos(x^2)\,dx$

③ $\int \frac{e^{\ln(x)}}{x}\,dx$

④ $\int \frac{\ln x}{x}\,dx$

⑤ $\int \frac{2x}{3x^2+4}\,dx$

⑥ $\int \frac{2x}{(5x^2+1)^3}\,dx$

Differentiate Answer

Less basic substitutions

① $\int \frac{x}{\sqrt{x+1}}\,dx$

② $\int \frac{x^3}{\sqrt{x^2-1}}\,dx = \int \frac{x}{\sqrt{x^2-1}}\cdot x^2\,dx$

$u = x^2-1$
$du = 2x$
$x^2 = u+1$

③ $\int \frac{2x+1}{x^2+1}\,dx$

④ $\int \sin^4 x \cdot \cos x\,dx$

⑤ $\int \tan x \sec^2 x\,dx$

⑥ $\int \frac{x^6}{x^{14}+1}\,dx$ (1 less than half)

⑦ $\int \frac{3}{x\sqrt{x^6-1}}\,dx$

warm-up

$$\int_{x=1}^{x=3} x(x^2+1)\,dx = ?$$

$$\boxed{u = x^2+1} \longrightarrow \begin{array}{l} x=1 \Rightarrow u = 1^2+1 = 2 \\ x=3 \Rightarrow u = 10 \end{array}$$

$$\frac{d}{dx}(u) = \frac{d}{dx}(x^2+1)$$

$$\frac{du}{dx} = 2x$$

$$du = 2x\,dx$$

$$\boxed{\frac{1}{2x}du = dx}$$

$$= \int_{\square}^{\square} x \cdot u \,\frac{1}{2x}du$$

$$= \frac{1}{2}\int_{2}^{10} u\,du$$

$$= \frac{1}{2}\left(\frac{u^2}{2}\right)\Bigg|_{2}^{10} = \frac{1}{2}\left[\frac{100}{2} - \frac{4}{2}\right] = \frac{1}{2}[50-2] = \frac{1}{2}[48] = 24$$

Substitution!

(warm-up)

$$\int_{x=1}^{x=3} x(x^2+1)^3\,dx =$$

$\boxed{u = x^2+1}$ $x=1 \Rightarrow u = 1^2+1 = 2$

$x=3 \Rightarrow u = 10$

$$\frac{d}{dx}(u) = \frac{d}{dx}(x^2+1)$$

$$\frac{du}{dx} = 2x \quad \text{so} \quad du = 2x\,dx$$

↑ treat like fraction

$$\boxed{\frac{1}{2x}du = dx}$$

seating chart (Friday wk 1)

Bernie

Brenan

Shane

Nicholas

Alex   Tyler

Ethan

Josh   Hudson   McKenzie

Jesse

Fact: $\int dx$ is linear //

$$= \int x(u)^3 \frac{1}{2x}du = \frac{1}{2}\int \cancel{x}\cdot u^3 \frac{1}{\cancel{x}}du$$

$$(\star) = \frac{1}{2}\int_2^{10} u^3\,du = \frac{1}{2}\frac{u^4}{4}$$

$$= \frac{u^4}{8}\Big|_2^{10} = \frac{10^4}{8} - \frac{2^4}{8}$$

$$= \frac{(9984)}{8}$$

$$= \frac{(x^2+1)^4}{8}\Big|_1^3$$

same