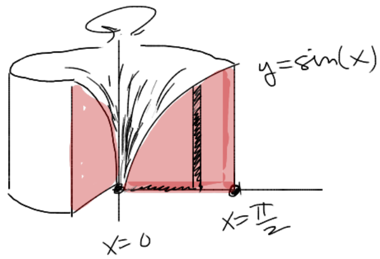
Wk 1 - Friday

More u-sub.

Application: Vol. of Solid of Rev. (y-axis)

Shell Method.



surface area of shell

$2\pi r h$

$2\pi \cdot x \cdot f(x)$

$$V=\int_0^{\pi/2} 2\pi x \cdot \sin(x)\,dx = 2\pi\int_0^{\pi/2} x\cdot \sin(x)\,dx = 2\pi\left[x\cos(x)+\overbrace{\int \cos(x)\,dx}^{\sin(x)}\Big|_0^{\pi/2}\right]$$

LIPET

$u=x \qquad dv=\sin(x)$

$du=dx \qquad v=-\cos(x)$

y-coord @ 90° = 1

$$=2\pi\left[-\frac{\pi}{2}\cdot\underbrace{\cos\frac{\pi}{2}}_{=0}+\sin\frac{\pi}{2}-\left[0\cdot\cos(0)+\sin(0)\right]\right]=2\pi$$

More I.B.P.

$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

① $\int \ln(\sqrt{x})\,dx = x\cdot\ln(\sqrt{x}) - \underbrace{\int \overbrace{\frac{1}{2x}\cdot x}^{\frac{1}{2}}\cdot dx}_{-\frac{1}{2}\int dx} = x\ln(\sqrt{x}) - \frac{1}{2}x + C$

$u = \ln(\sqrt{x}) \qquad dv = dx$

$= \boxed{x\left(\ln(\sqrt{x}) - \frac{1}{2}\right) + C}$

$du = \underbrace{\frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}}_{\frac{1}{2x}}\,dx \qquad v = x$

Check: $\frac{d}{dx}(\text{ans}) = 1\cdot\left(\ln(\sqrt{x}) - \frac{1}{2}\right) + \underbrace{x\left(\frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}\right)}_{+\frac{1}{2}} = \ln(\sqrt{x})$

②

$$\int \cos(\sqrt{x})\,dx = x\cdot\cos(\sqrt{x}) + \tfrac{1}{2}\int \sqrt{x}\,\sin(\sqrt{x})\,dx$$

Technique: (often used with sqrt(x) as argument) …
w-sub first, then I.B.P.

$u = \cos(\sqrt{x})$ $\quad dv = dx$ $\quad$ stuck! b/c this harder than OG

$du = -\sin(\sqrt{x})\cdot\frac{1}{2\sqrt{x}}$ $\quad v = x$

DEAD END

Idea! ~~u~~ w-sub 1st to transform into an easier I.B.P. problem

set
$w = \sqrt{x}$

$dw = \frac{1}{2\sqrt{x}}\,dx$

$2\sqrt{x}\,dw = dx$

$$= \int \cos(w)\cdot 2\sqrt{x}\,dw$$

familiar

$$= 2\int w\cdot\cos(w)\,dw = 2\Big(w\cdot\sin(w) \overbrace{- \int \sin(w)\,dw}^{+\cos(w)}\Big) = 2\big(w\cdot\sin(w) + \cos(w)\big)$$

$$= 2\big(\sqrt{x}\,\sin(\sqrt{x}) + \cos(\sqrt{x})\big)$$

Now I.B.P.:

$u = w$ $\quad dv = \cos(w)$

$du = dw$ $\quad v = \sin(w)$

check: $\frac{d}{dx}(\text{ans}) = 2\Big(\frac{1}{2\sqrt{x}}\cdot\sin(\sqrt{x}) + \sqrt{x}\cdot\cos(\sqrt{x})\cdot\frac{1}{2\sqrt{x}} - \sin(\sqrt{x})\frac{1}{2\sqrt{x}}\Big) = \cos(\sqrt{x})$

(3) $$\int \sqrt{x}\ln x\,dx = \frac{2}{3}x^{3/2}\cdot\ln(x) - \frac{2}{3}\underbrace{\int x^{1/2}\,dx}_{\frac{2}{3}x^{3/2}} = \frac{2}{3}x^{3/2}\left[\ln(x) - \frac{2}{3}\right] + c$$ LIPET

$u = \ln(x)$ $dv = \sqrt{x}\,dx$

$du = \frac{1}{x} = x^{-1}$ $v = \int x^{1/2}dx = \frac{2}{3}x^{3/2}$

check: $\frac{d}{dx}(\text{ans}) = x^{1/2}\left(\ln(x) - \frac{2}{3}\right) + \underbrace{\frac{2}{3}x^{3/2}\cdot\frac{1}{x}}_{\frac{2}{3}x^{1/2}} = \sqrt{x}\ln x$:)

(4) $$\int x\cdot\ln(\sqrt{x})\,dx = \int x\cdot\ln(w)\cdot 2w\,dw = \int w^2\ln(w)2w\,dw = 2\cdot\int w^3\ln(w)\,dw$$

w-sub 1st b/c $\sqrt{x}$ argument:

$\boxed{w = \sqrt{x}}$ $\rightsquigarrow$ $w^2 = x$

! $w^4 = x^2$

$dw = \frac{1}{2\sqrt{x}}dx$

$2\sqrt{x}\,dw = dx$

$\boxed{2w\,dw = dx}$

I.B.P.

$u = \ln(w)$ $dv = w^3\,dw$

$du = \frac{1}{w}dw$ $v = \int w^3 dw = \frac{w^4}{4}$

$= 2\left(\frac{w^4}{4}\cdot\ln(w) - \underbrace{\int\frac{w^4}{4}\cdot\frac{1}{w}\,dw}_{\frac{1}{4}\int w^3\,dw}\right)$

$= \frac{w^4}{2}\cdot\ln(w) - \frac{1}{2}\cdot\frac{w^4}{4} + c$

$= \boxed{\frac{x^2}{2}\cdot\ln(\sqrt{x}) - \frac{x^2}{8} + c}$

check: $\frac{d}{dx}(\text{ans}) = x\cdot\ln(\sqrt{x}) + \underbrace{\frac{x^2}{2}\cdot\frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}} - \frac{1}{4}x}_{=0} = x\ln(\sqrt{x})$:)

⑤

$$\int \sin^{-1}(x)\,dx = x\sin^{-1}x + \frac{1}{2}\int -2x \cdot \frac{1}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{w}}\,dw = \frac{1}{2}\int w^{-1/2}\,dw = \frac{2}{1}\cdot\frac{1}{2}\,w^{1/2} = \sqrt{w} = \sqrt{1-x^2}$$

$u = \sin^{-1}(x)$ $\quad dv = dx$ $\qquad w = (1-x^2)$

$du = \dfrac{1}{\sqrt{1-x^2}}\,dx$ $\quad v = x$ $\qquad dw = -2x\,dx$

$$= x\cdot\sin^{-1}x + \sqrt{1-x^2}$$

check $\frac{d}{dx}(\text{ans}) = 1\cdot\sin^{-1}x + \underbrace{x\cdot\frac{1}{\sqrt{1-x^2}} + \frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x)}_{=0}$:)