Wk 1 Fri

7-0. part II 11

$$\int \frac{1}{x\sqrt{100x^2 - 1}}\,dx =$$

$x > 0 \Rightarrow x = |x|$

want:

$u^2 = 100x^2$

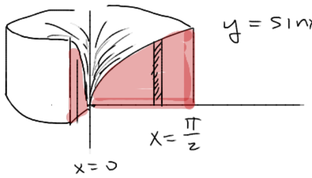
so set: $u = 10x$

$$\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{|x|\sqrt{x^2-1}}$$

often we drop abs value sign

More I.B.P.

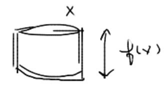
Find the volume of the solid of revolution:



about y-axis

Shell Method

$$V = \int_0^{\pi/2} 2\pi x f(x)\, dx$$



$S = 2\pi x f(x)$

$$V = \int_0^{\pi/2} 2\pi x \cdot \sin x\, dx = 2\pi \int x \cdot \sin x\, dx = 2\pi\left(-x\cos x + \int \cos x\, dx\right)$$

LIPET

$u = x \qquad dv = \sin x\, dx$

$du = dx \qquad v = \int dv = -\cos x$

$$= 2\pi\left(-x\cos x + \sin x\right)\Big|_0^{\pi/2} = 2\pi\left(-\frac{\pi}{2}\cdot \underbrace{\cos\frac{\pi}{2}}_{=0} + \underbrace{\sin\frac{\pi}{2}}_{=1} - 0 + \underbrace{\sin 0}_{=0}\right)$$

$$= 2\pi$$

① $\int \ln(\sqrt{x})\,dx = x\cdot\ln(\sqrt{x}) - \int \frac{1}{2x}\cdot x\,dx = x\cdot\ln(\sqrt{x}) - \frac{1}{2}x = x\left(\ln(\sqrt{x}) - \frac{1}{2}\right)$

$\sqrt{x} = x^{1/2} \rightarrow \frac{1}{2}x^{-1/2}$

$\frac{1}{2\sqrt{x}}$

$$\boxed{\begin{array}{ll} u = \ln(\sqrt{x}) & dv = dx \\ du = \frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}} = \frac{1}{2x} & v = \int dv = x \end{array}}$$

$-\frac{1}{2}\int dx = -\frac{1}{2}x$

check: $\frac{d}{dx}(\text{ans}) = 1\left(\ln(\sqrt{x}) - \frac{1}{2}\right) + x\Big(\underbrace{\frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}}_{\frac{1}{2x}}\Big) = \ln(\sqrt{x})$

② $\int \cos(\sqrt{x})\,dx = x\cdot\cos(\sqrt{x}) + \int \frac{\sin(\sqrt{x})}{2\sqrt{x}}\cdot x\,dx$

Technique: apply ~~u~~ w-sub first, to transform into an easier IBP problem.

$u = \cos(\sqrt{x})$ $\quad dv = dx$

$du = -\frac{\sin(\sqrt{x})}{2\sqrt{x}}\cdot dx$ $\quad v = x$

$\int \frac{1}{2}\sin(\sqrt{x})\cdot\sqrt{x}\,dx$

$\int \cos(\sqrt{x})\,dx = \int \cos(w)\cdot 2w\,dw = 2\int w\cdot\cos(w)\,dw = 2\Big[w\cdot\sin(w) - \overbrace{\int \sin(w)\,dw}^{+\cos(w)} = w\sin(w) + \cos(w)\Big]$

① $w = \sqrt{x}$

$dw = \frac{1}{2\sqrt{x}}dx$

$2w\,dw = 2\sqrt{x}\,dw = dx$

I.B.P. $\quad u = w \quad dv = \cos(w)\,dw$

$du = dw \quad v = \sin(w)$

$\boxed{2\left[\sqrt{x}\cdot\sin(\sqrt{x}) + \cos(\sqrt{x})\right]}$

check: $\frac{d}{dx}(\text{ans})\, 2\left[\frac{1}{2\sqrt{x}}\cdot\sin(\sqrt{x}) + \sqrt{x}\cos(\sqrt{x})\cdot\frac{1}{2\sqrt{x}} - \sin(\sqrt{x})\cdot\frac{1}{2\sqrt{x}}\right] = \cos(\sqrt{x})$

(use w-sub first)

③ $\int \ln(\sqrt{x})\,dx = \int \ln(w)\cdot 2w\,dw = 2\int w\cdot\ln(w)\,dw$

$\boxed{w = \sqrt{x}}$

$dw = \frac{1}{2\sqrt{x}}dx$

$2\sqrt{x}\,dw = dx$

$\boxed{2w\,dw = dx}$

Now I.B.P.

$u = \ln(w) \quad dv = w\,dw$

$du = \frac{1}{w}dw \quad v = \frac{w^2}{2}$

$= 2\left[\frac{w^2}{2}\cdot\ln(w) - \frac{1}{2}\int w\,dw\right] = w^2\ln(w) - \frac{w^2}{2} + c$

$= w^2\left(\ln(w) - \frac{1}{2}\right) + c$

$= x\left(\ln(\sqrt{x}) - \frac{1}{2}\right) + c$

④ $\int \sqrt{x}\ln x\,dx = \frac{2}{3}x^{3/2}\cdot\ln(x) - \frac{2}{3}\int x^{1/2}dx = \frac{2}{3}x^{3/2}\ln|x| - \frac{2}{3}\cdot\frac{2}{3}x^{3/2}$

$u = \ln(x) \quad dv = \sqrt{x}\,dx$

$du = \frac{1}{x}dx \quad v = \int\sqrt{x}\,dx = \frac{2}{3}x^{3/2}$

$= \frac{2}{3}x^{3/2}\left[\ln(x) - \frac{2}{3}\right]$

⑤ $\int x\ln(\sqrt{x})\,dx = \int x\ln(w)\,2w\,dw = 2\int w^3\cdot\ln(w)\,dw = 2\left[\frac{w^4}{4}\cdot\ln(w) - \frac{1}{4}\int w^3\,dw\right]$

$w = \sqrt{x} \quad w^2 = x$

$dw = \frac{1}{2\sqrt{x}}dx$

$2\sqrt{x}\,dw = dx$

$2w\,dw = dx$

$u = \ln(w) \quad dv = w^3$

$du = \frac{1}{w}dw \quad v = \frac{w^4}{4}$

$= x^2\cdot\ln(\sqrt{x}) - \frac{1}{2}\frac{w^4}{4} + c$

$\boxed{x^2\cdot\ln(\sqrt{x}) - \frac{1}{8}x^2 + c}$

$\frac{1}{2}x^2\left(\ln(\sqrt{x}) - \frac{1}{8}\right) + c$

check: $\frac{d}{dx}(\text{ans}) = x\left(\ln(\sqrt{x})\right) + \frac{1}{2}x^2\cdot\ln(\sqrt{x})\cdot\frac{1}{2\sqrt{x}}$