you need to be able to finish these in less than a minute . . . ..

1. $\int x\,dx = \frac{x^2}{2} + C$

2. $\int \sqrt[3]{x}\,dx = \int x^{1/3}\,dx = \frac{x^{4/3}}{4/3} + C = \frac{3}{4}\cdot x^{4/3} + C$

3. $\int \frac{1}{x}\,dx = \ln|x| + C$

4. $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = 2x^{1/2} + C$

5. $\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C$ — why? →

6. $\int \frac{1}{\sqrt{1-x^2}}\,dx =$

7. $\int \frac{1}{x\sqrt{x^2-1}}\,dx =$

8. $\int e^x\,dx =$

9. $\int \sin x\,dx =$

10. $\int \cos x\,dx =$

11. $\int \sec^2 x\,dx =$

12. $\int \sec x \tan x\,dx =$

Note: $(\ln(x))' = \frac{1}{x}\cdot 1 = \frac{1}{x}$

$\ln(-x) = \frac{1}{-x}\cdot(-1) = \frac{1}{x}$

Implicit Differentiation:

$\frac{d}{dx}(xy) = 1\cdot y + x\cdot\frac{dy}{dx} = 1\cdot y + xy'$ (product)

Pythag. Thg Id'

$a^2+b^2=c^2$

$\cos^2\theta + \sin^2\theta = 1$

$\div \cos^2\theta$

$1 + \tan^2\theta = \sec^2\theta$

$\frac{d}{dx}(\tan^{-1}x) = ?$

① set $y = \tan^{-1}x$

② $\frac{d}{dx}(y) = \frac{d}{dx}(\tan^{-1}x)$

②b $\tan(y) = x$

③ $\frac{d}{dx}(\tan(y)) = \frac{d}{dx}(x) = 1$

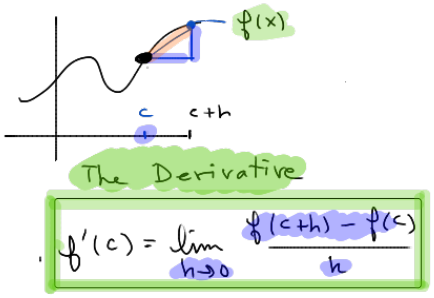
$\sec^2(y)\cdot\frac{dy}{dx}$

④ $\frac{dy}{dx} = \frac{1}{\sec^2(y)} = \frac{1}{1+\tan^2 y}$

$= \frac{1}{1+x^2}$

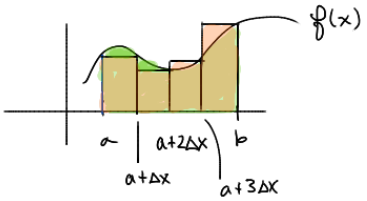
Thursday

- 1. Calculus I Review
  - a. What is Calculus I?
    - i. Precise way of calculating change (in functions)
    - ii. Two Branches: Differential & Integral
      1. Both depend on the idea of an infinite limit



Link: https://www.desmos.com/calculator/t6sxoyxyty

The Definite Integral



w/ $\Delta x = \frac{b-a}{n}$

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x^*) \cdot \Delta x$$

Link: https://www.desmos.com/calculator/tgyr42ezjq

Note! Differentiating is "easier" than integrating

$$\frac{d}{dx}\left(e^{u}\right) = e^{u} \cdot \frac{du}{dx}$$

eg:

$$\frac{d}{dx}\left(e^{x^2}\right) = e^{x^2} \cdot 2x$$

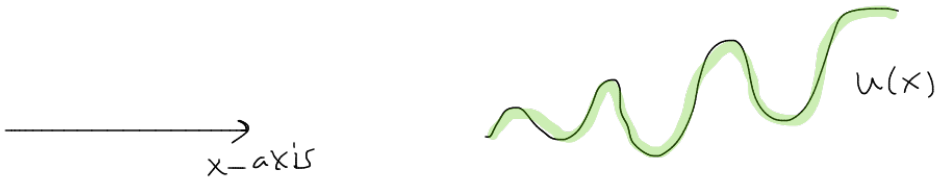
$$\int e^{x^2}\,dx = \text{NO elementary sol'n}$$

unless our function is given discrete data

u-sub

In this fact: $\int x^n dx = \frac{x^{n+1}}{n+1}$

the only requirement is that x be continuously varying real variable



True that $\int u^n du = \frac{u^{n+1}}{n+1}$ where $u = f(x)$

Ex. $\int (3x^2+1)^5 \cdot x\, dx$

Note: there's no product rule for integration.

Idea: Partition integrand into two pieces where one is (a multiple of) the derivative of the other.

$u = 3x^2+1$

$\frac{d}{dx}(u) = \frac{d}{dx}(3x^2+1)$

$\frac{du}{dx} = 6x$ so

treat like fraction

$du = 6x\,dx$

$\frac{1}{6x} \cdot du = dx$

$= \int u^5 \times \frac{1}{6x} du = \frac{1}{6}\int u^5 du$

$= \frac{1}{6} \cdot \frac{u^6}{6} + C$

$= \frac{(3x^2+1)^6}{36} + C$