wk 1 — thur

Integration by parts

recall, the product rule for derivatives

$$(f \cdot g)' = f' \cdot g + f g'$$

isolate

$$f g' = (f \cdot g)' - f' g$$

integrate

$$\int f g' = \int (f \cdot g)' - \int f' g = f \cdot g - \int g \cdot f'$$

w/ $u = f$

$v = g$ ...

$$\int u v' = u \cdot v - \int v \cdot u'$$

w/ du instead of u' ultra violet minus $\int$uper voo doo

$$\boxed{\int u\,dv = u \cdot v - \int v\,du}$$

Use I.B.P. to integrate products

choose ______ so that (1) du is more simple than u

(2) $\int dv$ should be no harder than the OG

(3) use EVERYTHING behind $\int$

- put dx w/ dv

$$\int u\,dv = uv - \int v\,du$$

(4) Hint for choosing u: LIPET — log, inv. trig, power, exp, trig

Ex

$$\int xe^x\,dx$$

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = \int dv = \int e^x dx = e^x$

$$= x \cdot e^x - \int e^x dx = xe^x - e^x + C$$

$$= e^x(x-1) + C$$

check:

$$\frac{d}{dx}(\text{ans}) = e^x(x-1) + e^x \cdot 1 = e^x x - e^x + e^x = xe^x$$

:)

Ex

$$\int x \cdot \cos(x)\,dx = x \cdot \sin(x) - \int \sin(x)\,dx = \boxed{x \cdot \sin(x) + \cos(x) + C}$$

$\int$ Stuff dx

$u = St \qquad dv = uff\,dx$

recognize integrand is a product ⇒ I.B.P.

$u = x \qquad dv = \cos(x)\,dx$

$du = dx \qquad v = \int \cos(x)\,dx = \sin(x)$

check: $\frac{d}{dx}(\text{ans}) = 1 \cdot \sin x + x \cdot \cos x - \sin x = x \cdot \cos x$

Ex

$$-\frac{1}{2}\int x\,dx = -\frac{1}{2} \cdot \frac{x^2}{2} = \frac{-x^2}{4}$$

$$\int x \cdot \ln(x)\,dx = \frac{x^2}{2} \cdot \ln(x) - \int \frac{x^2}{2} \cdot \frac{1}{x}\,dx = \frac{x^2}{2} \cdot \ln(x) - \frac{x^2}{4} = \boxed{\frac{x^2}{2}\left[\ln(x) - \frac{1}{2}\right]}$$

$u = \ln(x) \qquad dv = x\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{x^2}{2}$

$u = x \qquad dv = \ln(x)\,dx$

$du = dx \qquad v = \int \ln(x)\,dx$

poor choice b/c this is harder than OG!

$$\frac{d}{dx}(\text{an}) = \frac{2x}{2}\left[\ln(x) - \frac{1}{2}\right] + \frac{x^2}{2}\left[\frac{1}{x}\right] = x \cdot \ln x - \frac{x}{2} + \frac{x}{2} = x \cdot \ln x$$

:)

$\int x^2 e^x dx =$

$u = x^2 \quad dv = e^x dx$

$du = 2x\, dx \quad v = e^x$

$u = 2x \quad dv = e^x dx$

$du = 2 dx \quad v = e^x$

$x^2 e^x - \underbrace{\int e^x \cdot 2x\, dx} = x^2 e^x - 2x e^x + 2e^x$

$-\left[2xe^x - \underbrace{\int e^x \cdot 2 dx}_{2e^x}\right]$

$= e^x\left[x^2 - 2x + 2\right]$

$\frac{d}{dx}(ans) = e^x(x^2 - 2x + 2) + e^x(2x - 2) = x^2 e^x$:)

$\int x^3 e^x dx$ multiple I.B.P's

$\int x^{10} e^x dx$

Technique: Apply I.B.P. multiple times

$$\int \ln(x)\,dx = x\cdot\ln(x) - \int x\cdot\frac{1}{x}\,dx = x\cdot\ln x - \int dx = x\ln(x) - x + C$$

$$= \boxed{x\,(\ln(x)-1) + C}$$

$u = \ln(x) \quad dv = dx$

$du = \frac{1}{x}\,dx \quad v = \int 1\,dx = x$

Check: $\frac{d}{dx}\left(x(\ln(x)-1)\right) = 1(\ln(x)-1) + x\left(\frac{1}{x}\right) = \ln x - 1 + 1 = \ln x$ ☺

technique: set u = EVERYTHING except dx

technique: apply I.B.P. twice, revealing the O.G. integral, then isolate it.

Ex

LIPET

$$\int e^x \cdot \cos x \, dx = e^x \cdot \sin(x) - \int e^x \sin(x)\, dx$$

$u = e^x \qquad dv = \cos x \, dx$

$du = e^x dx \qquad v = \int dv = \sin(x)$

I.B.P again

$u = e^x \qquad dv = \sin x \, dx$

$du = e^x dx \qquad v = -\cos x$

$$-\left[-e^x \cos x + \int e^x \cos x \, dx\right]$$

$$\int e^x \cos x \, dx = e^x \sin(x) + e^x \cos x - \int e^x \cos x \, dx$$

gather

$$2\int e^x \cos x \, dx = e^x(\sin x + \cos x) \Rightarrow \int e^x \cos x \, dx = \frac{1}{2} e^x (\sin(x) + \cos(x))$$