MA163 wk 1 Thurs

Recall the product rule for derivatives $(fg)' = f'g + fg'$

or w/ $u$ & $v$:

$$(uv)' = u'v + v'u$$

isolate

$$uv' = (u\cdot v)' - u'v$$

integrate:

anti-der, derivative

$$\int uv' = \int (u\cdot v)' - \int u'v$$

$$\boxed{\int uv' = u\cdot v - \int vu'}$$

more common to use $d$ for derivative: product rule

$$d(uv) = du\cdot v + u\cdot dv$$

isolate this

$$u\cdot dv = d(uv) - v\cdot du$$

integrate

$$\int u\,dv = \int d(uv) - \int v\,du$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Integration by parts -

Remember the right hand side: Ultra - Violet minus Super Voo Doo

This is the main technique to integrate products

"product rule for integrals"

Ex

$\int x \cdot e^x\,dx = x \cdot e^x - \int e^x\,dx = \boxed{x \cdot e^x - e^x + C}$

recognize: the integrand is a product $\Rightarrow$ think I.B.P.

Everything behind $\int$ (including $dx$!) is a part.

$u = x$ $\quad dv = e^x\,dx$

derivative $\quad$ integrate

$du = dx$ $\quad v = \int dv = e^x$

| Hint: choose $u$ so that $du$ is more simple |
|---|

check! $\frac{d}{dx}(\text{ans}) = 1 \cdot e^x + x e^x - e^x = x e^x$ :)

$\int \text{stuff}\,dx = uv - \int v\,du$

$u = \text{stuf}$ $\quad dv = f\,dx$

$du =$ $\quad v =$

| Helpful Hint for choosing $u$: LIPET |
|---|

L: log; I: inverse trig; P: power; E: exponentials; T: trig

Ex $\int x \cdot \cos(x)\,dx$

$u = x$ $\quad dv = \cos(x)\,dx$

$du = dx$ $\quad v = \sin(x)$

$= x \cdot \sin(x) - \int \sin(x)\,dx = x \cdot \sin(x) + \cos(x) + C$

check: $\frac{d}{dx}(\text{ans}) = 1 \cdot \sin x + x\cos x - \sin x$

$= x \cdot \cos x$

Ex $\int x \cdot \ln(x)\,dx$

$u = x$ $\quad dv = \ln(x)\,dx$

$dv = dx$ $\quad v = \int \ln(x)\,dx$

$\leftarrow$ this is what it looks like to make a wrong guess

stuck

$u = \ln(x)$ $\quad dv = x\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = \int dv = \frac{x^2}{2}$

$= \frac{x^2}{2} \cdot \ln(x) - \underbrace{\int \frac{x^2}{2} \cdot \frac{1}{x}\,dx}$

$-\int \frac{x}{2}\,dx$

$-\frac{x^2}{4}$ $\Rightarrow$ $\frac{x^2}{2} \cdot \ln(x) - \frac{x^2}{4} + c = \frac{x^2}{2}\left[\ln(x) - \frac{1}{2}\right] + C$

check: $\frac{d}{dx}(\text{ans}) = x\left(\ln(x) - \frac{1}{2}\right) + \frac{x^2}{2}\left(\frac{1}{x}\right) = x \ln x$

Technique: Perform IBP multiple times

$$\int x^2 e^x \, dx = e^x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} e^x \, dx$$

$u = e^x \qquad dv = x^2 \, dv$

$du = e^x \, dx \qquad v = \int x^2 \, dv = \frac{x^3}{3}$

stuck

similarly:

$$\int x^3 e^x \, dx$$

$$\int x^5 e^x \, dx$$

$u = x^2 \qquad dv = e^x \, dx$

$du = 2x \, dx \qquad v = e^x$

$$= x^2 \cdot e^x - \underbrace{\int e^x \cdot 2x \, dx}$$

$u = 2x \qquad dv = e^x \, dx$

$du = 2 \, dx \qquad v = e^x$

$$= 2xe^x - \underbrace{\int e^x \cdot 2 \, dx}_{-2e^x}$$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

$$= e^x (x^2 - 2x + 2) + C$$

Technique: set u = EVERYTHING` except dx

$$\int \ln(x)\,dx \qquad u = \ln(x) \quad dv = dx \qquad du = \frac{1}{x} \quad v = \int dx = x$$

$$= x \cdot \ln(x) - \int \frac{1}{x} \cdot x \, dx$$

$$x \cdot \ln(x) - \underbrace{\int dx}_{x} = x \ln x - x + c = x(\ln(x) - 1) + c$$

check: $\frac{d}{dx}(\text{ans}) = 1(\ln(x) - 1) + x\left(\frac{1}{x}\right) = \ln x - 1 + 1 = \ln x$