

MA163 wk 1 Thurs

Recall the product rule for derivatives  $(fg)' = f'g + fg'$

or w/  $u \frac{1}{x} v$ :

isolate  $(uv)' = u'v + v'u$

integrate:  $uv' = (u \cdot v)' - u'v$

$\int uv' = \int (u \cdot v)' - \int u'v$   
*anti-der*  $\int (u \cdot v)$  *derivative*

$\int uv' = u \cdot v - \int v u'$

more common to use  $d$  for derivative:  
product rule  
 $d(uv) = du \cdot v + u \cdot dv$

isolate this

$u \cdot dv = d(uv) - v \cdot du$   
↓ integrate

$\int u \, dv = \int d(uv) - \int v \, du$

$\int u \, dv = uv - \int v \, du$

Integration by parts-

Remember the right hand side: Ultra - Violet minus Super Voo Doo

this is the main technique to integrate products  
"product rule for integrals"

Ex  $\int x \cdot e^x dx = x \cdot e^x - \int e^x dx = x \cdot e^x - e^x + C$

recognize: the integrand is a product  $\Rightarrow$  think I.B.P.  
Everything behind  $\int$  (including  $dx!$ ) is a part.

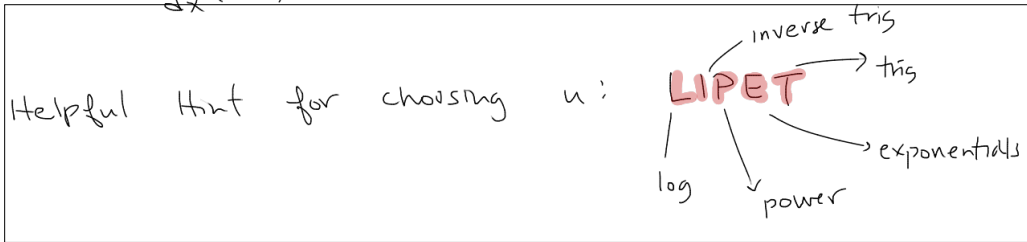
$u = x$   
derivative  
 $du = dx$

$dv = e^x dx$   
integrate  
 $v = \int dv = e^x$

Hint: choose  $u$  so that  $du$  is more simple

$\int \text{stuff } dx = uv - \int v du$   
 $u = \text{stuff}$   $dv = f dx$   
 $du =$   $v =$

check:  $\frac{d}{dx}(\text{ans}) = 1 \cdot e^x + x e^x - e^x = x e^x$  ✓



Ex  $\int x \cdot \cos(x) dx$

$u = x$   $dv = \cos(x) dx$   
 $du = dx$   $v = \sin(x)$

check:  
 $\frac{d}{dx}(\text{ans}) = 1 \cdot \sin(x) + x \cos(x) - \sin(x) = x \cdot \cos(x)$

$x \cdot \sin(x) - \int \sin(x) dx = x \cdot \sin(x) + \cos(x) + C$

Ex  $\int x \cdot \ln(x) dx$

$u = x$   $dv = \ln(x) dx$   
 $du = dx$   $v = \int \ln(x) dx$

this is what it looks like  
 $\leftarrow$  to make a wrong guess  
**stuck**

$\frac{x^2}{2} \cdot \ln(x) - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$   
 $-\int \frac{x}{2} dx$   
 $-\frac{x^2}{4}$

$u = \ln(x)$   $dv = x dx$   
 $du = \frac{1}{x} dx$   $v = \int dv = \frac{x^2}{2}$

$\Rightarrow \frac{x^2}{2} \cdot \ln(x) - \frac{x^2}{4} + C = \frac{x^2}{2} \left[ \ln(x) - \frac{1}{2} \right] + C$

check:  $\frac{d}{dx}(\text{ans}) = x \left( \ln(x) - \frac{1}{2} \right) + \frac{x^2}{2} \left( \frac{1}{x} \right) = x \ln(x)$

$$\int x^2 e^x dx = e^x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} e^x dx$$

$$u = e^x \quad dv = x^2 dx$$
$$du = e^x dx \quad v = \int x^2 dx = \frac{x^3}{3}$$

stuck

similarly:

$$\int x^3 e^x dx$$

$$\int x^5 e^x dx$$

$$u = x^2 \quad dv = e^x dx$$

$$du = 2x dx \quad v = e^x$$

$$= x^2 \cdot e^x - \int e^x \cdot 2x dx$$

$$u = 2x \quad dv = e^x dx$$

$$du = 2 dx \quad v = e^x$$

$$= 2xe^x - \int e^x \cdot 2 dx$$

$-2e^x$

$$= x^2 e^x - 2xe^x + 2e^x + c$$

$$= e^x(x^2 - 2x + 2) + c$$

Technique: Perform IBP multiple times

Technique: set  $u = \text{EVERYTHING}$  except  $dx$

$$\int \ln(x) dx$$

$$u = \ln(x) \quad dv = dx$$

$$du = \frac{1}{x} \quad v = \int dx = x$$

$$x \cdot \ln(x) - \int \frac{1}{x} \cdot x dx$$

$$x \cdot \ln(x) - \int \frac{dx}{x} = x \ln x - x + c = x(\ln(x) - 1) + c$$

check:  $\frac{d}{dx}(\text{ans}) = 1(\ln(x) - 1) + x\left(\frac{1}{x}\right) = \ln x - 1 + 1 = \ln x$  😊