wk 1 - Wed

$$\begin{array}{c} \text{Integration} \to \text{Numerical} \\ \text{Integration} \to \text{Chart} \\ \text{Integration} \to \text{Substitution} \\ \text{Integration} \to \text{I.B.P.} \end{array}$$

$\int \frac{1}{u}\,du$

CHART

Functions		Anti-Derivatives
u^n	$n \neq -1$	$\frac{u^{n+1}}{n+1}$
$\frac{1}{u}$		$\ln\lvert u\rvert$
e^u		e^u
$\sin(u)$		$-\cos(u)$
$\cos(u)$		$\sin(u)$
$\sec^2(u)$		$\tan(u)$
$\sec(u)\tan(u)$		$\sec(u)$
$\csc^2(u)$		$-\cot(u)$
$\csc(u)\cot(u)$		$-\csc(u)$

Functions	Anti-Derivatives
$\frac{1}{1+u^2}$	$\tan^{-1}(u)$
$\frac{1}{\sqrt{1-u^2}}$	$\sin^{-1}(u)$
$\frac{1}{\lvert u\rvert\sqrt{u^2-1}}$	$\sec^{-1}(u)$

Basic Substitutions

① $\int e^{7x+1}\,dx =$

① $u = 7x+1$ ③ $du = 7\,dx$

② $\frac{d}{dx}(u) = \frac{du}{dx} = 7$ ④ $\boxed{\frac{1}{7}\,du = dx}$

$\int e^u\,du$

$\int e^u \cdot \frac{1}{7}\,du = \frac{1}{7}\int e^u\,du = \boxed{\frac{1}{7}e^{7x+1} + C}$

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②

$\int 3x\cos(x^2)\,dx = \frac{1}{2}3\int \cos(x^2)\cdot \underbrace{2x\,dx}_{du} = \frac{3}{2}\int \cos(u)\,du = \boxed{\frac{3}{2}\sin(x^2) + C}$

$u = x^2$

$du = \underbrace{2x\,dx}_{\because}$

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③ $\int \frac{e^{\ln(x)}}{x}\,dx \xrightarrow{\text{or}} \int e^{\ln(x)}\cdot\frac{1}{x}\,dx = \int \frac{e^u}{\cancel{x}}\cdot \cancel{x}\,du = \int e^u\,du$

$\int e^u\,du$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\Rightarrow x\,du = dx$

Less Basic Sub

$$\int \frac{x}{\sqrt{x+1}}\,dx = \int \frac{x}{(x+1)^{1/2}}\,dx \qquad u = x+1 \quad \frac{du}{dx} = 1 \quad du = dx$$

rewrite $\sqrt{\ } = (\ )^{1/2}$, set $u =$ inside ( )

$$= \int \frac{x}{(u)^{1/2}}\,du$$

stuck b/c mix of x & u | mine u! $\longrightarrow$ $(u-1 = x)$

①

$$\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$

$$= \int \frac{u-1}{u^{1/2}}\,du = \int \frac{u}{u^{1/2}} - \frac{1}{u^{1/2}}\,du$$

$$= \int u^{1/2} - u^{-1/2}\,du = \frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2} + c = \boxed{\frac{2}{3}\cdot(x+1)^{3/2} - 2(x+1)^{1/2} + c}$$

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② mixed var $\Rightarrow$ stuck

$$\int \frac{x^3}{\sqrt{x^2-1}}\,dx = \int \frac{x^3}{u^{1/2}}\,\frac{1}{2x}\,du = \frac{1}{2}\int \frac{x^2}{u^{1/2}}\,du$$

$\boxed{u = x^2 - 1} \rightsquigarrow u + 1 = x^2$

$du = 2x\,dx$

$\boxed{\frac{1}{2x}du = dx}$

$$= \frac{1}{2}\int \frac{u+1}{u^{1/2}}\,du = \frac{1}{2}\int u^{\frac{1}{2}} + u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{2}\left[\frac{2}{3}u^{3/2} + 2u^{1/2} + c\right] = \frac{1}{3}(x^2-1)^{3/2} + (x^2-1)^{1/2} + c$$

③

$$\int \frac{2x+1}{x^2+1}\,dx = \int \frac{2x}{x^2+1} + \frac{1}{x^2+1}\,dx = \int \frac{1}{u}\,du + \int \frac{1}{x^2+1}\,dx$$

$u = x^2 + 1$
$du = 2x\,dx$

?

$$\boxed{\ln|x^2+1| + \tan^{-1}(x) + c}$$

④

$$\int \frac{1}{1+u^2}\,du$$

$$\int \frac{x^6}{x^{14}+1}\,dx = \int \frac{x^6}{1 + x^{14}}\,dx = \int \frac{x^6}{1+u^2}\,\frac{1}{7x^6}\,du$$

set $u^2 = x^{14}$ | $du = 7x^6\,dx$

$\Rightarrow u = x^7$ | $\frac{1}{7x^6}du = dx$

$$= \frac{1}{7}\tan^{-1}(u) + c = \frac{1}{7}\tan^{-1}(x^7) + c$$