Wk 1 — Wed

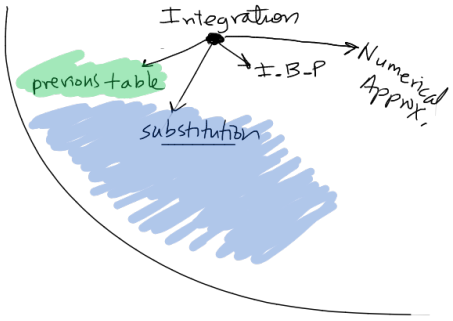
Basic Anti-Derivatives

Functions	Anti-Derivatives
u^n $\quad n \neq -1$	$\frac{u^{n+1}}{n+1}$
$\frac{1}{u}$	$\ln\lvert u\rvert$
$\sin(u)$	$-\cos(u)$
$\cos(u)$	$\sin(u)$
$\sec^2(u)$	$\tan(u)$
$\sec(u)\tan(u)$	$\sec(u)$
$\csc^2(u)$	$-\cot(u)$
$\csc(u)\cot(u)$	$-\csc(u)$
e^u	e^u

$$\int e^u \, du = e^u + C$$

Functions	Anti-Derivatives
$\frac{1}{1+u^2}$	$\tan^{-1}(u)$
$\frac{1}{\sqrt{1-u^2}}$	$\sin^{-1}(u)$
$\frac{1}{\lvert u\rvert\sqrt{u^2-1}}$	$\sec^{-1}(u)$

Basic Substitution Integrals



1. $\int e^{7x+1}\,dx$

$u = 7x+1$ | $du = 7\,dx$

$\frac{d}{dx}(u) = \frac{du}{dx} = 7$ | $\frac{1}{7}\,du = dx$

$\int e^u \frac{1}{7}\,du = \frac{1}{7}\underbrace{\int e^u\,du}_{\text{chart}} = \frac{1}{7}e^u + C = \boxed{\frac{1}{7}e^{7x+1} + C}$

2. $\int 3x\cdot\cos(x^2)\,dx = \int \cos(x^2)\,3x\,dx = 3\int \cos(x^2)\,x\,dx = \frac{3}{2}\int \cos(x^2)\,2x\,dx$

think $\int \cos(u)\,du$ ... $2x$

want 2 here

$= \frac{3}{2}\int \cos(u)\,du$

$= \frac{3}{2}\sin(x^2) + C$

CIE $\Rightarrow \int e^u\,du$

3. $\int \frac{e^{\ln(x)}}{x}\,dx = \int e^u \cdot \frac{1}{x}\,dx = \int e^u\,du = \boxed{e^{\ln(x)} + C}$

$u = \ln(x)$ | $du = \frac{1}{x}\,dx$

4. $\int \frac{\ln(x)}{x}\,dx = \int \frac{u}{x}\cdot \cancel{x}\,du = \int u\,du = \frac{(\ln(x))^2}{2} + C$

$u = \ln(x)$ | $\cancel{x}\,du = dx$

$du = \frac{1}{x}\,dx$

Less Basic Substitutions

$$\int \frac{x}{\sqrt{x+1}}\,dx = \int \frac{x}{(x+1)^{1/2}}\,du$$

b/c rel. is simple!

$u = x+1 \dashrightarrow u-1 = x$

$du = dx$

$\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$

(1)

$$= \int \frac{x}{u^{1/2}}\,du = \int \frac{u-1}{u^{1/2}}\,du = \int \frac{u}{u^{1/2}} - \frac{1}{u^{1/2}}\,du$$

$$= \int u^{1/2} - u^{-1/2}\,du$$

$\frac{2}{3}u^{3/2} - 2u^{1/2} + C$

$\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + C$

(2)

hint:

$u = x^2 - 1 \rightsquigarrow u+1 = x^2$

$du = 2x\,dx$

$\frac{1}{2x}\cdot du = dx$

$$\int \frac{x^3}{\sqrt{x^2-1}}\,dx$$

$$= \int \frac{x^3}{u^{1/2}} \cdot \frac{1}{2x}\,du$$

$$= \frac{1}{2}\int \frac{x^2}{u^{1/2}}\,du \quad (x^2 \leftarrow \text{trouble})$$

$$= \frac{1}{2}\int \frac{u+1}{u^{1/2}}\,du = \frac{1}{2}\left[\int u^{1/2} + u^{-1/2}\,du = \frac{1}{2}\left[\frac{2}{3}\cdot u^{3/2} + 2u^{1/2}\right]\right.$$

$$= \frac{1}{3}(x^2-1)^{3/2} + (x^2-1)^{1/2} + C$$

(3)

$$\frac{1}{7}\int \frac{7x^6}{x^{14}+1}\,dx$$

$u = x^7$

$du = 7x^6\,dx$

$$= \boxed{\frac{1}{7}\tan^{-1}(x^7) + C}$$

no obvious u!

inv. trig? $\frac{1}{1+u^2} \Rightarrow$ $u^2 = x^{14}$ or $u = x^7$