Anti-Derivatives for TRIG

$$\int \sin(u)\,du = -\cos(u)$$

$$\int \cos(u)\,du = \sin(u)$$

$$\int \sec(u)\tan(u)\,du = \sec(u)$$

$$\int \tan(u) = -\ln|\cos(u)|$$

$$\int \csc(u)\cot(u)\,du = -\csc(u)$$

$$\int \cot(u) = \ln|\sin(u)|$$

$$\int \tan(u)\,du = \int \frac{\sin(u)}{\cos(u)}\,du$$

set $w = \cos(u)$

$dw = -\sin(u)\,du$

$\frac{-1}{\sin(u)}\,dw = du$

$$\int \frac{dx}{x} = \int x^{-1}\,dx$$

$\ln|x|$

$$= \int \frac{\sin(u)}{w} \cdot \frac{-1}{\sin(u)}\,dw = -\int \frac{dw}{w} = -\ln|w| + C$$

$$= \boxed{-\ln|\cos(u)| + C}$$

$\frac{d}{dx}(\ln u) = \frac{1}{u}\cdot\frac{du}{dx} = \frac{du/dx}{u}$

check: $\frac{d}{du}\left(-\ln(\cos(u)) + C\right) = -\frac{(-\sin(u))}{\cos(u)}$

$= \frac{\sin(u)}{\cos(u)} = \tan(u)$ ✓

$$\int \cot(u)\,du = \int \frac{\cos(u)}{\sin(u)}\,du$$

$w = \sin(u)$

$dw = \cos(u)\,du$

$$= \int \frac{dw}{w} = \ln|\sin(u)| + C$$

Ex.

$$\int 3x \cdot \sin(x^2+1)\,dx = 3\int \sin(x^2+1)\cdot x\,dx = 3\int \sin(u)\left(\tfrac{1}{2}\right)du = \tfrac{3}{2}\int \sin(u)\,du$$

$x\,dx = \frac{1}{2}du$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$\frac{1}{2}du = x\,dx$

$\int 3x \cdot \sin(u)\,\frac{1}{2x}\,du$

$\frac{1}{2x}\,du = dx$

$= \boxed{-\frac{3}{2}\cos(x^2+1)+C}$

check

$\frac{d}{dx}(\text{ans}) = -\frac{3}{2}\left(-\sin(x^2+1)\cdot 2x\right) = \boxed{3x\cdot\sin(x^2+1)+C}$

Ex

$$\int 5x^4 \sec(x^5)\tan(x^5)\,dx = \int \sec(x^5)\tan(x^5)\,5x^4\,dx$$

$u = x^5$

$du = 5x^4\,dx$

$= \int \sec(u)\tan(u)\,du = \sec(u)+C$

$= \boxed{\sec(x^5)+C}$