Basic u-sub

① $\int (3x^2+1)\, x\, dx$

$u = 3x^2 + 1$

$du = 6x\, dx$ $\quad \Big| \quad dx = \frac{1}{6x}\, du$

$\int (u)\, x \cdot \frac{1}{6x}\, du$

$= \frac{1}{6}\int u\, du$

② $\int (\sin x)^2 \cdot \cos x\, dx$

$u = \sin x$

$du = \cos x\, dx$

$\int u^2\, du$

③ $\int e^{x^2} \cdot x\, dx$

$u = e^{x^2}$

$du = e^{x^2} \cdot 2x\, dx$ } dead end

$u = x^2$

$du = 2x$

the key for u-sub is to find a subset of the problem whose derivative also appears.

---

④ $\frac{1}{6}\int \frac{6 \cdot 3x^5}{\sqrt{x^6+7}}\, dx$

$u = x^6+7$

$du = 6x^5\, dx$

$= \frac{3}{6}\int \frac{du}{u^{1/2}} = \boxed{\frac{1}{2}\int u^{-1/2}\, du} = \frac{2}{1}\cdot\frac{1}{2}\, u^{\frac{1}{2}}$

$= \boxed{\sqrt{x^6+7} + C}$

⑤ $\int \frac{\cos x}{\sin x}\, dx$

$u = \sin x$

$\boxed{\ln|\sin x| + C}$

$\frac{1}{x \ln x} = \frac{1}{(x^{-1})^{-1} \ln x} = \frac{x^{-1}}{\ln x} = \frac{\frac{1}{x}}{\ln x}$

⑥ $\int \frac{1}{x \ln x}\, dx$

$u = \ln x$

$du = \frac{1}{x}\, dx$

$\int \frac{1}{x} \cdot \frac{1}{\ln x}\, dx$

$\int \frac{\frac{1}{x}}{\ln x}\, dx$

$\int \frac{du}{u} = \ln|u|$

$= \ln|\ln|x||$

---

2 $\int \frac{e^{\cot x} \cdot \csc x}{\sin x}\, dx$

$\boxed{u = \cot x}$

$du = -\csc^2 x\, dx$

$\boxed{\frac{1}{-\csc^2 x}\, du = dx}$

$\csc x = \frac{1}{\sin x}$

$= \int \frac{e^u \cdot \csc x}{\sin x} \cdot \frac{1}{-\csc^2 x}\, du = \int \frac{e^u}{\sin x \cdot (-\csc x)}\, du = -\int e^u\, du = \boxed{-e^{\cot x} + C}$