Thursday - Week 9

u-substitution ($u = f(x)$)

$$\int u^n \, du = \frac{u^{n+1}}{n+1}$$

Ex: $\int (3x+1)^5 \, dx = \int (u)^5 \, \frac{1}{3} du = \frac{1}{3}\int u^5 \, du \overset{\text{integrate}}{=} \frac{1}{6}\cdot\frac{1}{3} u^b + c = \boxed{\frac{1}{18}(3x+1)^6 + C}$

$u = 3x+1$, $n = 5$

$\frac{du}{dx} = 3$ so $du = 3\,dx$

$\frac{1}{3} du = dx$

check: $\frac{d}{dx}(\text{ans}) = \frac{6}{18}(3x+1)^5 \cdot 3 = (3x+1)^5$ ☺

Ex: $\int \sqrt{5x+1} \, dx = \int (u)^{\frac{1}{2}} \cdot \frac{1}{5} du = \frac{1}{5}\int u^{\frac{1}{2}} du = \frac{1}{5}\cdot\frac{2}{3} u^{\frac{3}{2}} + c = \boxed{\frac{2}{15}(5x+1)^{\frac{3}{2}} + C}$

$u = 1/2$

$u = 5x+1$

$\frac{du}{dx} = 5$, $du = 5\,dx$, $\frac{1}{5} du = dx$

check: $\frac{d}{dx}(\text{ans}) = \frac{3}{2}\cdot\frac{2}{15}(5x+1)^{\frac{1}{2}} \cdot 5 = (5x+1)^{1/2}$ ✓

$\int \frac{1}{x} dx = \ln|x|$

Ex: $\int \frac{1}{(4x+1)} dx = \int (4x+1)^{-1} dx = \frac{1}{4}\int (4x+1)^{-1} \cdot 4 \, dx = \frac{1}{4}\int u^{-1} du$

$u = 4x+1$

$du = 4dx$

↑ no power rule!

$= \frac{1}{4}\ln|u| + c$

$= \boxed{\frac{1}{4}\ln|4x+1| + c}$

check: $\frac{d}{dx}(\text{ans}) = \frac{1}{4}\cdot\frac{4}{4x+1} = \frac{1}{4x+1}$ ✓

Ex: $\int x^{99}\cdot(x^{100}+3)^2 \, dx = \int u^2 \, du$

$\int x^2 \, dx$ ↓ indicates x = variable

Look for "derivative relationships" .... "degree 1 differences"

$u = (\text{larger degree term}) = \boxed{x^{100}+3 = u}$

$\frac{du}{dx} = 100x^{99}$, $du = 100x^{99}\,dx$,

$\boxed{\frac{1}{100x^{99}} du = dx}$

sub. $= \int x^{99}(u)^2 \cdot \frac{1}{100x^{99}} du = \frac{1}{100}\int u^2 \, du = \frac{1}{3}\cdot\frac{1}{100} u^3 + c$

$= \boxed{\frac{(x^{100}+3)^3}{300} + c}$

check: $\frac{d}{dx}(\text{ans}) = \frac{3(x^{100}+3)^2}{300}\cdot 100x^{99} = x^{99}(x^{100}+3)^2$ ✓

Thursday - Week 9

u-substitution — (Integrals)

Power Rule

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C$$

Ex: $\int (3x+5)^4\,dx = \int (u)^4 \cdot \frac{1}{3}\,du = \frac{1}{3}\int u^4\,du = \frac{1}{5}\cdot\frac{1}{3}u^5 = \boxed{\frac{(3x+5)^5}{15} + C}$

Think: $\int u^4\,du$, ① $\boxed{u = 3x+5}$ (u = inside parenthesis)

② $\frac{du}{dx} = 3$, $du = 3\,dx$ or $\boxed{\frac{1}{3}\,du = dx}$

check: $\frac{d}{dx}(\text{ans}) = \frac{5}{15}(3x+5)^4 \cdot 3 = (3x+5)^4$ ✓

Ex $\int \sqrt{5x+7}\,dx = \int u^{\frac{1}{2}}\,du$

we integrate ↑

$\boxed{u = 5x+7}$, $\frac{du}{dx} = 5$, $\boxed{\frac{1}{5}\,du = dx}$

$= \int u^{\frac{1}{2}} \cdot \frac{1}{5}\,du = \frac{1}{5}\int u^{\frac{1}{2}}\,du = \frac{2}{3}\cdot\frac{1}{5}u^{\frac{3}{2}} + C = \boxed{\frac{2}{15}(5x+7)^{3/2} + C}$

check: $\frac{d}{dx}(\text{ans}) = \frac{3}{2}\cdot\frac{2}{15}(5x+7)^{1/2}\cdot 5 = (5x+7)^{1/2}$ ✓

Ex $\int \frac{1}{(5x+6)}\,dx = \int \frac{1}{u}\cdot\frac{1}{5}\,du = \frac{1}{5}\int \frac{1}{u}\,du = \frac{1}{5}\int \frac{du}{u} = \frac{1}{5}\ln|u| + C = \boxed{\frac{1}{5}\ln|5x+6| + C}$

$\boxed{u = 5x+6}$

$du = 5\,dx$

$\boxed{\frac{1}{5}\,du = dx}$

check: $\frac{d}{dx}(\text{ans}) = \frac{1}{5}\cdot\frac{5}{5x+6} = \frac{1}{5x+6}$ ✓

Ex. $\boxed{\int x^{99}\left(x^{100}+17\right)^3\,dx}$

key: recognize derivative relationships … degree one differences

set u = higher degree term

$\boxed{u = x^{100}+17}$

$\frac{du}{dx} = 100x^{99}$, $du = 100x^{99}\,dx$

$\boxed{\frac{1}{100x^{99}}\,du = dx}$

$= \int x^{99}(u)^3 \cdot \frac{1}{100x^{99}}\,du = \frac{1}{100}\int u^3\,du$

$= \frac{1}{400}u^4 + C = \boxed{\frac{1}{400}\left(x^{100}+17\right)^4 + C}$

$\frac{d}{dx}(\text{ans}) = \frac{4}{400}\left(x^{100}+17\right)^3 \cdot 100x^{99} = x^{99}\left(x^{100}+17\right)^3$ ✓