Estimation Practice

1. Find the value of n which will give an estimate of the definite integral below to within 0.001 of the actual value using the indicated method.

$$\int_0^5 \sin(x^2)\,dx$$

You can use:

$f(x) = \sin(x^2)$

$f'(x) = 2x\cos(x^2)$

$f''(x) = 2\cos(x^2) - 4x^2\sin(x^2)$

$$\begin{aligned} f^{(3)}(x) &= -4x\sin(x^2) - 8x\sin(x^2) - 8x^3\cos(x^2) \\ &= -12x\sin(x^2) - 8x^3\cos(x^2) \end{aligned}$$

$$\begin{aligned} f^{(4)}(x) &= -12\sin(x^2) - 24x^2\cos(x^2) - 24x^2\cos(x^2) + 16x^4\sin(x^2) \\ &= -12\sin(x^2) - 48x^2\cos(x^2) + 16x^4\sin(x^2) \end{aligned}$$

a. Trapezoidal

b. Midpoint

c. Simpson's Rule

2. Use integrals to estimate the series below to within 0.001 of the actual value.

$$\sum_{n=1}^{+\infty} \frac{1}{n^5}$$

3. Show that the following series converges by the Alternating Series Test. Then estimate the value of the series within 0.001 of the actual value.

$$\sum_{n=1}^{+\infty} \frac{(-1)^n}{n^5}$$

4. Estimate the given indefintie integral using the indicated method . . .

$$\int_0^4 x^2\,dx$$

(a) Using Trapezoidal Rule with $n = 2$.

$$S_{T,2} = 2\left[\frac{f(0)+f(2)}{2}\right] + 2\left[\frac{f(2)+f(4)}{2}\right] =$$

(b) Using Midpoint Rule with $n = 2$.

$$S_{M,2} = 2f(1) + 2f(3) =$$

(c) Using Simpson's Rule with $n = 4$.

$$S_{S,4} = \frac{1}{3}\left[f(0) + 4f(1) + 2f(2) + 4f(3) + f(4)\right] =$$

also calculate it this way:

$$S_{S,4} = \frac{1}{3}S_{T,2} + \frac{2}{3}S_{M,2} =$$

(d) Now just calculate the value directly:

$$\int_0^4 x^2\,dx =$$