1. More Taylor & Maclaurin stuff . . . .

(a) Find the interval of convergece for the power series below:

$$\alpha(x) = x^2 - \frac{1}{3!}x^5 + \frac{1}{6!}x^8 - \cdots = \sum_{n=0}^{+\infty} \frac{(-1)^n}{(3n)!} x^{3n+2}$$

$$\frac{\left|a_{n+1}\right|}{\left|a_n\right|} = \frac{\left|\frac{x^{n+1}}{(n+1)!}\right|}{\left|\frac{x^n}{n!}\right|} = \frac{n!}{(n+1)n!} \cdot \frac{|x|^{n+1}}{|x|^n} = \frac{1}{n+1}|x|$$

$$\lim_{n\to+\infty} \frac{\left|a_{n+1}\right|}{\left|a_n\right|} = \lim_{n\to+\infty} \frac{1}{n+1}|x| = |x| \lim_{n\to+\infty} \frac{1}{n+1} = |x|{\cdot}0 = 0 < 1$$

By the Ratio Test, this series converges for all values of x.

(b) Find the limit:

$$\lim_{x\to 0} \frac{\alpha(x)}{5x^2}$$

(c) Use a fifth degree polynomial to estimate

$$\int_0^1 \alpha(x)\, dx$$

2. Find the interval of convergence for the power series below:

$$\beta(x) = 1 - \frac{1}{2}(x-5) + \frac{1}{2\cdot 2^2}(x-5)^2 - \frac{1}{3\cdot 2^3}(x-5)^3 + \cdots$$
$$= 1 + \sum_{n=1}^{+\infty} \frac{(-1)^n}{n2^n}(x-5)^n$$

$$\frac{|a_{n+1}|}{|a_n|} = \frac{\left|\frac{(-1)^{(n+1)+1}}{n+1}x^{n+1}\right|}{\left|\frac{(-1)^{n+1}}{n}x^n\right|} = \frac{n}{n+1}\cdot\frac{|x|^{n+1}}{|x|^n} = \frac{n}{n+1}|x|$$

$$\lim_{n\to+\infty}\frac{|a_{n+1}|}{|a_n|} = \lim_{n\to+\infty}\frac{n}{n+1}|x| = |x|\lim_{n\to+\infty}\frac{n}{n+1} = |x|\cdot 1 = |x|$$

From the Ratio Test, we know that the series converges for valees of x such that $|x| < 1$, or $-1 < x < 1$ and diverges for $|x| > 1$, or $(-\infty, -1) \cup (1, +\infty)$. We have to check whether or not the series converges for $x = 1$ and $x = -1$.

When $x = 1$, the power series is equal to

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

which converges by the Alternating Series Test.

When $x = -1$, the power series is equal to

$$-1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \cdots = -\sum_{n=1}^{+\infty}\frac{1}{n}$$

which diverges by p-series test.

Therefore, the interval of convergence for the Maclaurin Series for $\ln(x + 1$ is $[-1, 1]$.