Math 163 - Calculus - Exam 3 - Guide **Name:** ____________________
March 25, 2025
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1. Find the third degree Taylor polynomial for $x^{3/2}$ about $x = 1$.

2. Find the arc length of the curve

$$y = \frac{2}{3}x^{3/2} \text{ from } 0 \text{ to } 3$$

3. The Maclaurin series for $\cos x$ is below.

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

Find the interval of convergence.

4. Use a series to approximate $\sin(x)$ to within $10^{-3}$ accuracy.

5. Use an eighth degree Taylor polynomial to estimate

$$\int_0^1 \frac{\cos(x^2) - 1}{x}\, dx =$$

6. The Maclaruin series for the funciton $\ln(x+1)$ is below. Find the interval of convergence.

$$x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots = \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n} x^n$$

7. Use the expansion above to find a series that converges to $\ln 2$.

8. Prove the following (most beautiful) equation is true:

$$e^{i\pi} + 1 = 0.$$

Hint: Find Maclaurin series for $e^x, \cos(x)$ and $\sin(x)$. Then evaluate the series for $e^x$ at $x = i\theta$. Finally, evaluate this series at $\theta = \pi$.