MacLaurin series for $\sin(x)$
$(x=0)$

$f(x) = \sin x$ @ $x=0$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^{iv}(x) = \sin x$

$f(0) = 0$

$f'(0) = 1$

$f''(0) = 0$

$f'''(0) = -1$

$f^{iv}(0) = 0$

$f^{(k)} = (-1)^k$

$$\boxed{\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$$

Monday - Week 10

warm-up:

1. watch video
2. Give the Maclaurin series for cos(x). Derive it,

$a=0$ in Taylor's formula: $\displaystyle\sum_{k=0}^{+\infty} \frac{f^{(k)}(a)\cdot(x-a)^k}{k!}$

$\cos(x) =$

you are finding the infinite polynomial represents cos(x).

	set $x=a=0$
$f(x) = \cos(x)$	$f(0) = 1$ ✓
$f'(x) = -\sin(x)$	$f'(0) = 0$ ✓
$f''(x) = -\cos(x)$	$f''(0) = -1$ ✓
$f'''(x) = \sin(x)$	$f'''(0) = 0$ ✓
$f^{(iv)}(x) = \cos(x)$	$f^{(iv)}(0) = 1$ ✓
pattern repeats --	1, 0, -1, 0, 1, ...

putting together

$$\cos(x) = 1\cdot\frac{(x)^0}{0!} + \underbrace{\frac{0\cdot x^1}{1!}}_{=0} - \frac{1x^2}{2!} + \underbrace{\frac{0\cdot x^3}{3!}}_{=0} + \frac{1\cdot x^4}{4!}$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$= \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

In summary

$$\boxed{\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots}$$

Maclaurin Series

$\frac{d}{dx}\downarrow$

$$-\sin(x) = 0 - \frac{2x}{2!} + \frac{4x^3}{4!} - \frac{6x^5}{6!} + \frac{8x^7}{8!}$$

So,...

$$\boxed{\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}}$$

In light of these formulas recall:

1. Even/odd

$f(-x) = -f(x)$

$f(-x) = f(x)$

eg, $f(x) = 5x^3 - x$ odd

cos(x)	— even function
sin(x)	— odd

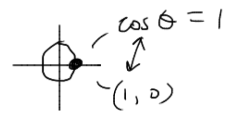
2. $\displaystyle\lim_{x\to 0} \frac{\sin(x)}{x} \underset{\text{near } x=0}{\approx} \frac{x}{x} = 1$

The Most beautiful equation in all of mathematics —

$$e^{i\pi} + 1 = 0$$

relates the top 5 most important numbers —.

Reminder:



$i = \sqrt{-1}$.

$i^2 = -1$

$i^3 = i \cdot i^2 = i(-1) = -i$

$i^4 = i^2 i^2 = (-1)(-1) = 1$

$\cos(0)$

$\sin(0)$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

In your homework,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \dots$$

$$e^{ix} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!} + \dots$$

$$= 1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} - \frac{x^6}{6!} + \dots$$

$$= \cos x + i \sin x$$

sub $\pi = x$

$$e^{i\pi} = \underbrace{\cos\pi}_{=-1} + \underbrace{i\sin\pi}_{=0}$$

So $e^{i\pi} = -1$

$$\boxed{e^{i\pi} + 1 = 0}$$