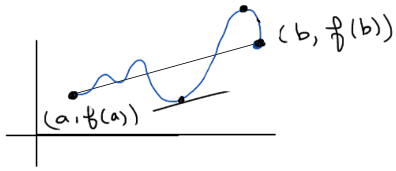
Week 10

Arc Length:

uses: Mean Value theorem



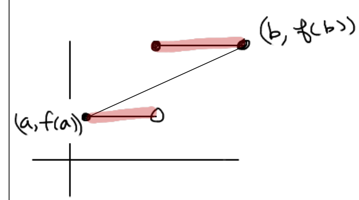
The average rate of change $= \frac{f(b)-f(a)}{b-a}$

If $f(x)$ is continuous then

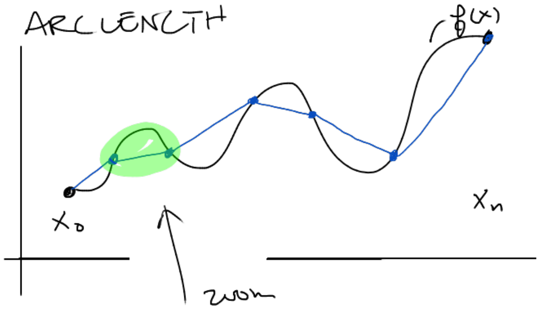
$\exists\ c \in (a,b)$ s.t.

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

when $f(x)$ is not cts



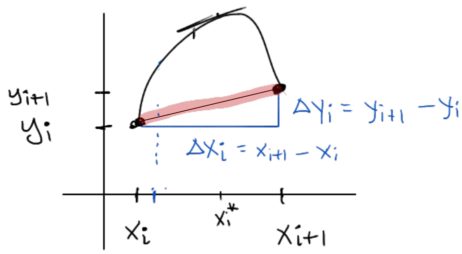
ARCLENGTH



What is length of this curve?

Ans: We'll answer using 'the philosophy of calculus'

A 'calculus' way of solving problems is to break complicated problems up into simpler ones, approximate and repeat.



Total Length ≈ sum of Intermediate segment lengths

$$\approx \sum_{i=0}^{N} l_i$$

w/ $l_i = \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$

$$= \sqrt{(\Delta x_i)^2 \left(1 + \frac{(\Delta y_i)^2}{(\Delta x_i)^2}\right)}$$

$$= \Delta x_i \cdot \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2}$$

$$= \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2} \cdot \Delta x_i$$

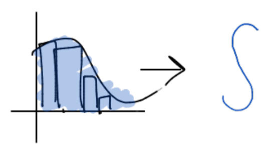
$$\text{Total Length} \approx \sum_{i=0}^{N} \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2} \cdot \Delta x_i$$

Mean value theorem

$$\text{Total Length} = \underbrace{\lim_{N \to \infty} \sum_{i=0}^{N}}\sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2} \cdot \Delta x_i = \lim_{N \to \infty} \sum_{i=0}^{N} \sqrt{1 + (f'(x_i^*))^2}\ \Delta x_i$$

"avg. slope"

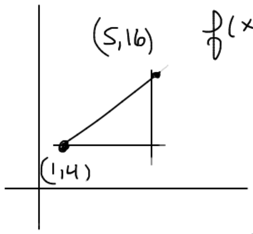
the "c" from the M.V.T.



Arc Length Formula	$\int_a^b \sqrt{1 + (f'(x))^2}\, dx$

Simple Ex.

Compute length of $f(x) = 3x+1$ b/w $(1,4)$ & $(5,16)$



$$\text{Arc length} = \int_1^5 \sqrt{1+(3)^2}\,dx$$

$$= \int_1^5 \sqrt{10}\,dx$$

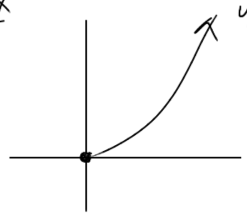
$$= \sqrt{10}\cdot x \Big|_1^5 = 5\sqrt{10} - 1\cdot\sqrt{10}$$

$$= 4\sqrt{10}$$

dist formula $= \sqrt{(5-1)^2 + (16-4)^2}$

$$= \sqrt{4^2+12^2} = \sqrt{160} = \sqrt{16\cdot 10} = 4\sqrt{10}$$

Ex $y = x^2$ from $x=0$, to $x=4$.



$$\int_0^4 \sqrt{1+(2x)^2}\,dx = \int_0^4 \sqrt{1+4x^2}\,dx =$$

$\frac{d}{dx}(x^2) = 2x$

$\int \sqrt{a^2+u^2}$

$a = 1$

$u = 2x$

$\to$ tan