MA 163 Exam 3 Guide

1. Find the third degree Taylor polynomial for $x^{3/2}$ about $x = 1$.

2. Use integrals to estimate the series below to within 0.001 of the actual value.

$$\sum_{n=4}^{+\infty} \frac{1}{4n^3}$$

3. The Maclaurin series for $\cos x$ is below.

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

Find the interval of convergence.

4. Use the series above to obtain an estimate for $\cos\left(\frac{1}{2}\right)$ to within 0.01 of the actual vallue.

5. Use an eighth degree Taylor polynomial to estimate

$$\int_0^1 \frac{\cos(x^2) - 1}{x}\, dx =$$

6. The Maclaruin series for the funciton $\ln(x+1)$ is below. Find the interval of convergence.

$$x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots = \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n} x^n$$

7. Use the expansion above to find a series that converges to $\ln 2$.

8. Prove the following (most beautiful) equation is true:

$$e^{i\pi} + 1 = 0.$$

Hint: Find Maclaurin series for $e^x, \cos(x)$ and $\sin(x)$. Then evaluate the series for e^x at $x = i\theta$. Finally, evaluate this series at $\theta = \pi$.