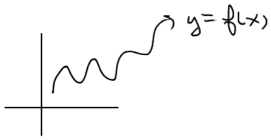
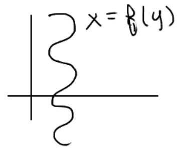
thur: wk 11

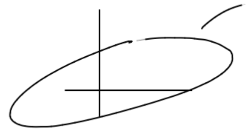
<u>Exam 4</u>: take home

Parametric Curve: curve defined by an additional variable

what you've seen!



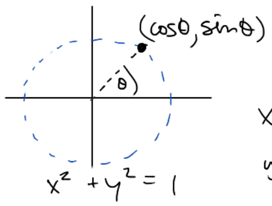




circles, ellipses, etc are parametric.

$x = f(t)$

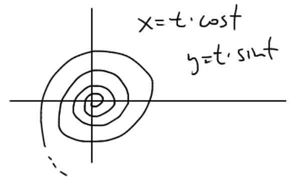
$y = f(t)$



$(\cos(23), \sin(23))$

$x(t) = \cos(t)$

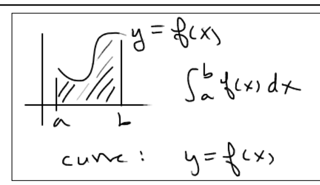
$y(t) = \sin(t)$

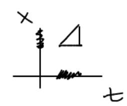


$x = t\cdot\cos t$

$y = t\cdot\sin t$

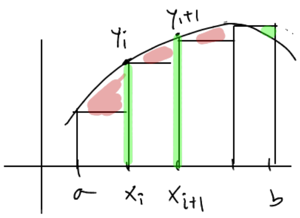
Recall Area under curve:





goal: produce a formula for the area under a parametric curve.

secant slope $-\frac{\Delta x}{\Delta t} \to \frac{dx}{dt}$ $\quad \Delta t \to dt$



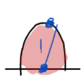
$$\text{Approx Area} = \sum \text{height}\cdot\text{width}$$

$$= \sum_{i=0}^{N} y_i\,(x_{i+1}-x_i)\cdot\frac{t_{i+1}-t_i}{t_{i+1}-t_i} = \sum \underset{\downarrow\, y}{y_i}\,\underbrace{\frac{x_{i+1}-x_i}{t_{i+1}-t_i}}_{x'(t)}\cdot(t_{i+1}-t_i)$$

$$=$$

$$\text{Exact Area} = \lim_{N\to\infty} \Rightarrow \quad \text{Area under parametric curve} = \int_a^b y(t)\cdot x'(t)\,dt$$

<u>Ex</u> Compute the area of a semi-circle w/ radius = 1. $\quad y=\sqrt{1-x^2}$



$(\cos(t), \sin(t))$ where $0 \le t \le \pi$

$$\int_{-1}^{1}\sqrt{1-x^2}\,dx \quad \text{or} \quad \int_1^{-1} -\sqrt{1-x^2}\,dx$$

$$A = \int_a^b y(t)\,x'(t)\,dt$$

$$= \int_\pi^0 \sin(t)\cdot(-\sin(t))\,dt = -\int_\pi^0 \sin^2(t)$$

Half-angle formula

$dt = \frac{1}{2}du$

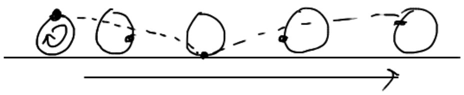
$u = 2t,\ du = 2dt$

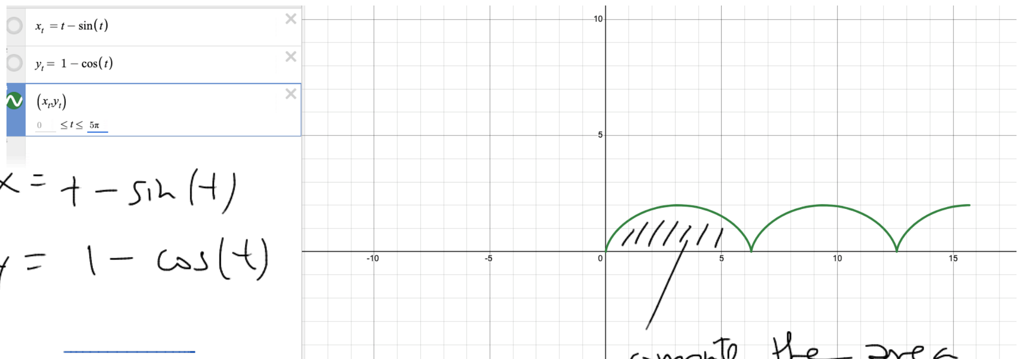
$t=\pi \Rightarrow u = 2\pi,\ t=0 \Rightarrow u=0$

$$= -\int_\pi^0 \frac{1-\cos(2t)}{2} = -\int_\pi^0 \frac{1}{2}\,dt + \frac{1}{2}\int_\pi^0 \cos(2t)\,dt = \underbrace{-\int_\pi^0 \frac{1}{2}\,dt}\, + \frac{1}{4}\int_{2\pi}^0 \cos(u)\,du$$

$$= -\frac{1}{2}t\Big|_\pi^0 + \underbrace{\frac{1}{4}\sin(u)\Big|_{2\pi}^0}_{=0} = \boxed{\frac{\pi}{2}}$$

cycloid





$x = t - \sin(t)$

$y = 1 - \cos(t)$

compute the area.

$$A = \int_0^{2\pi} (1-\cos(t))(1-\cos(t))\,dt = \int_0^{2\pi} (\underbrace{1-\cos(t))^2}_{\text{FOIL}}\,dt$$

$$= \int 1 - 2\cos(t) + \cos^2(t)\,dt$$

$$\int 1\,dt - 2\int \cos(t)\,dt + \int \cos^2(t)\,dt$$

$$\int_0^{2\pi} 1\,dt - 2\underbrace{\int_0^{2\pi} \cos(t)\,dt}_{\sin(t)\big|_0^{2\pi} = 0} + \int_0^{2\pi} \frac{1+\cos(2t)}{2}\,dt$$

$$\frac{1}{4}\sin(4\pi)\Big|_0^{2\pi} = 0$$

$$= 2\pi + 0 + \int_0^{2\pi} \frac{1}{2}\,dt + \overbrace{\int_0^{2\pi} \frac{1}{2}\cos(2t)\,dt}^{=0}$$

$$\frac{1}{2}[2\pi - 0] = \pi$$

$$= 3\pi$$