arc length, surface area, & center of mass

1. Find the arc length . . . .

(a)

$$x = \frac{y^4}{8} + \frac{1}{4y^2}, 1 \leq y \leq 2$$

$$\int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = \frac{33}{16}$$

(b)

$$y = \ln(\sec x), 0 \leq x \leq \frac{\pi}{4}$$

$$\int_0^{\pi/4} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \ln\left(1 + \sqrt{2}\right)$$

2. Find the surface area of the revolution . . . .

(a) Skip - moved to take home test

$$y = \sin \pi x, 0 \leq x \leq 1, \text{ rotate about the } x - \text{axis}$$

$$\int_0^1 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = ??$$

(Use trig sub with $\tan\theta = \sin \pi x$ . . . .)

(b)

$$y = 1 - x^2, 0 \leq x \leq 1, \text{ rotate about the } y - \text{axis}$$

$$\int_0^1 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \frac{5\pi}{6}\sqrt{5} - \frac{\pi}{6}$$

3. Find the center of mass of the lamina in the first quadrant enclosed by the curves $y = \sqrt[3]{x}$ and $y = x^3$.

$$M = \int_0^1 \int_{x^3}^{\sqrt[3]{x}} 1\, dy\, dx = \frac{1}{2}$$

$$M_y = \int_0^1 \int_{x^3}^{\sqrt[3]{x}} x\, dy\, dx = \frac{8}{35}$$

$$M_x = \int_0^1 \int_{x^3}^{\sqrt[3]{x}} y\, dy\, dx = \frac{8}{35}$$

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{M}, \frac{M_x}{M}\right) = \left(\frac{16}{35}, \frac{16}{35}\right)$$