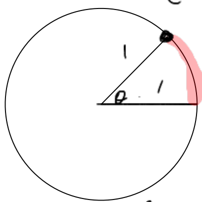
Thursday – Week 12

Warm-up: $\cos\theta$ = x-coord for term. pt of θ

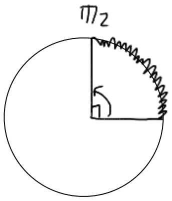
(cos θ, sin θ)



— what is this arc length?

θ above, in radians

arc length $= \theta$



Circumference

$C = 2\pi r$

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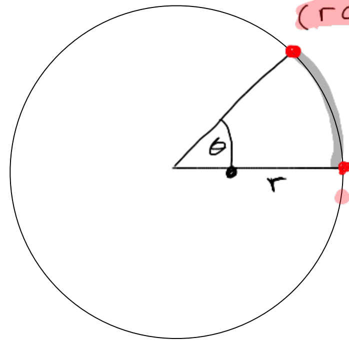
(r cos θ, r sin θ)



— arc length

← arc is a function of x.

radius = r

Hint! $x^2 + y^2 = r^2 \longleftrightarrow$ circle of radius r

So... top half of circle: $y = \sqrt{r^2 - x^2}$

Arc length Formula $= \int_a^b \sqrt{1 + \left[\frac{dy}{dx}\right]^2}\, dx$

$$\frac{dy}{dx} = \frac{1}{2}\left[r^2 - x^2\right]^{-1/2} \cdot \left[-2x\right] = \frac{-x}{\sqrt{r^2 - x^2}} \qquad \left[\frac{dy}{dx}\right]^2 = \frac{x^2}{r^2 - x^2}$$

$$L = \int_{\underbrace{r\cos\theta}_{\text{lower bound on } x}}^{\overbrace{r}^{\text{upper bound on } x}} \sqrt{\underset{\uparrow \; \frac{r^2 - x^2}{r^2 - x^2}}{1} + \frac{x^2}{r^2 - x^2}}\, dx = \int_{r\cos\theta}^{r} \sqrt{\frac{r^2}{r^2 - x^2}}\, dx = r\int_{r\cos\theta}^{r} \frac{1}{\sqrt{r^2 - x^2}}\, dx$$

$$= (-1) r \int_{r\cos\theta}^{r} \frac{-1}{\sqrt{r^2 - x^2}}\, dx = -r\left[\cos^{-1}\left(\frac{x}{r}\right)\right]\Bigg|_{r\cos\theta}^{r} = -r\left[\cos^{-1}\left(\frac{r}{r}\right) - \cos^{-1}\left(\frac{r\cos\theta}{r}\right)\right]$$

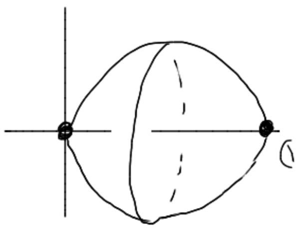
$$= -r\left[\cos^{-1}(1) - \cos^{-1}(\cos\theta)\right] = -r\left[0 - \theta\right] = r\theta$$

Surface Area of surface of revolution:

$y = \sin(\pi x) \qquad 0 \le x \le 1 \qquad$ revolve about x-axis.

$\frac{dy}{dx} = \pi \cos(\pi x)$

$$A = \int_0^1 2\pi \cdot y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$



$$= \int_0^1 2\pi \cdot \sin(\pi x) \sqrt{1 + (\pi \cos(\pi x))^2}\, dx$$

$$= 2\pi \int_0^1 \sqrt{1 + (\pi \cdot \cos(\pi x))^2} \cdot \sin(\pi x)\, dx$$

Idea!

$$\int \sqrt{1 + u^2} \cdot du$$

$u = \tan\theta$

$\boxed{\text{sub: } \tan\theta = \pi \cdot \cos(\pi x)}$

differentials

$$\sec^2\theta \cdot d\theta = -\pi^2 \sin(\pi x)\, dx$$

$$\left(\frac{1}{-\pi^2}\right) \sec^2\theta\, d\theta = \sin(\pi x)\, dx$$

$$= 2\pi \int_0^1 \sqrt{1 + \tan^2\theta} \left(\frac{-1}{\pi^2}\right) \sec^2\theta\, d\theta$$

$$= 2\pi \int_0^1 \sqrt{\sec^2\theta} \left(\frac{-1}{\pi^2}\right) \sec^2\theta\, d\theta$$

$$= \frac{-2}{\pi} \int_0^1 \sec^3\theta\, d\theta$$

Break into $\sec^2\theta \cdot \sec\theta$, substitute $\tan\theta^2 + 1$,

Integration by parts when it seems your stuck.