wk 12 thur

Differential Equations:

equations that involve an unknown variable (y), an indepent variable x — and one or more derivatives of y.

Ex $\frac{dT}{dt} = k(A-T)$, $\frac{dy}{dx} = 1 - 6e^{2x}$, $\frac{dy}{dx} = y \cdot x$

$\frac{dy}{dx} = 1 - 6e^{2x}$: type: directly integrable

Common Task: solve the D.E. (Find the function whose derivative satisfies the D.E.)

- General Solution: A family of functions all whose derivatives match eq'n
- Particular Solution: Single sol'n that satisfies D.E. plus an "initial condition"

Ex Find the general soln to

(a) $\frac{dy}{dx} = 1 - 6e^{2x}$

$dy = 1 - 6e^{2x}\,dx$

$\int 1\,dy = \int 1 - 6e^{2x}\,dx$

$y + C_1 = x - 6 \cdot \frac{1}{2} e^{2x} + C_R$

$y = x - 3e^{2x} + C$ ← gen'l sol

particular soln

$\boxed{y = x - 3e^{2x} + 8}$

(b) Find the particular soln satisfying $y(0) = 5$.

$5 = y(0) = 0 - 3e^{2\cdot 0} + C \Rightarrow 5 = -3 + C \Rightarrow C = 8$

Separable Differential Eq'ns

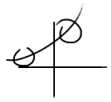
· Def'n Separable D.E. of $\frac{dy}{dx} = f(x) \cdot g(y)$ (multiplication)

then: multiply by dx, divide by $g(y)$

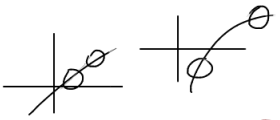
$$\frac{1}{g(y)}\,dy = f(x)\,dx$$

LHS: all y — RHS: all x

integrate both sides wrt their variables



EX $y' = y \cdot x$ goal: find the function y whose derivative = itself times x



1. Recognize separated.

$$\frac{1}{y} \cdot y' = x$$

(a)

$$\frac{1}{y}\frac{dy}{dx} = x \Rightarrow \frac{1}{y} \cdot dy = x \cdot dx \Rightarrow \int \frac{1}{y}\,dy = \int x\,dx$$

$$\ln|y| = \frac{x^2}{2} + C$$

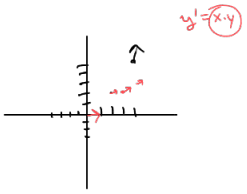
$$e^{\ln|y|} = e^{\frac{x^2}{2}+C}$$

$$|y| = e^{\frac{x^2}{2}} \cdot e^{C} = Ae^{\frac{x^2}{2}}$$

$$\boxed{y = Ae^{\frac{x^2}{2}}}$$

Rename $e^C = A$

A = constant



(b) Solve the I.V.P

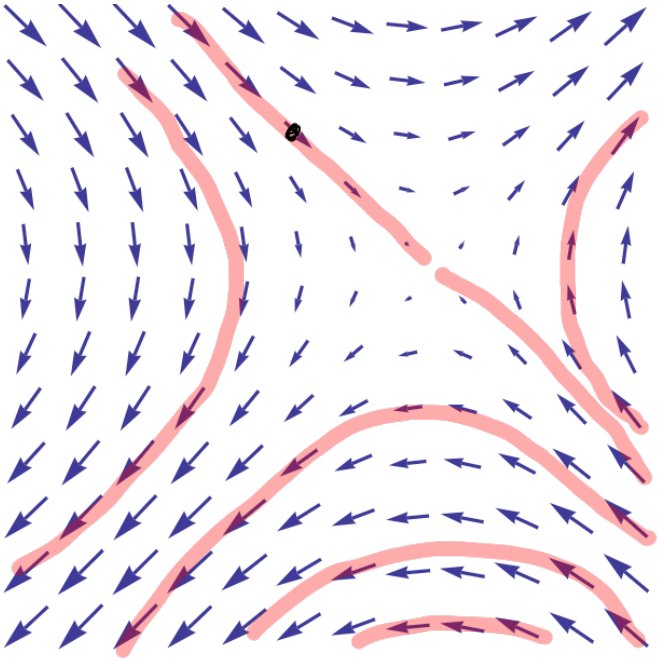
$y' = yx$, $y(0) = -3$

$y(0) = Ae^0 \Rightarrow A = -3$

$y(0) = -3$

$y = -3e^{\frac{x^2}{2}}$

Diffy Q's determine vector fields, solns are paths thru the field target @ each pt.



Solve the I.V.P.

$$y \cdot y' = x e^{-y^2} \quad , \quad y(0) = -5$$

$$e^{y^2} \cdot y \cdot y' = x$$

$$e^{y^2} \cdot y \cdot \frac{dy}{dx} = x$$

$$\int e^{y^2} \cdot y \, dy = \int x \, dx$$

$$\frac{1}{2} e^{y^2} = \frac{x^2}{2} + C$$

Isolate y:

$$e^{y^2} = x^2 + 2C = x^2 + A$$

$$y^2 = \ln(x^2 + A)$$

$$y = \pm \sqrt{\ln(x^2 + A)}$$

gen'l soln

(sub $y(0) = 5$

$$5 = \pm \sqrt{\ln(0 + A)} = \pm \sqrt{\ln(A)}$$

$5 > 0 \Rightarrow$ choose + ... $5 = \sqrt{\ln(A)}$

$$25 = \ln A$$

$$e^{25} = A$$

Particular soln $y = \pm \sqrt{\ln(x^2 + e^{25})}$