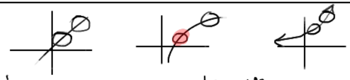
wk 12 – Thur

Differential Equations:



what is a D.E.? An eqn involving an unknown variable, y & its derivatives

$$\frac{dy}{dx} = y \cdot x \quad , \quad \frac{dy}{dx} = 1 - 6e^{2x} \quad , \quad y' \cdot y = x + 7 \quad , \quad y'x = x^2(y^2+1)$$

these are "1st order" or "degree 1" b/c it involves only the 1st derivative

Your task: solve a D.E. (i.e., find y, the function whose derivative is described by the D.E.)

Ex $\frac{dy}{dx} = 1 - 6e^{2x}$ this one is "directly integrable".

(1) Find all possible solutions "general sol'n"

$$\frac{dy}{dx} = 1 - 6e^{2x} \implies y = \int 1 - 6e^{2x}\, dx = \int 1\, dx - \int 6e^{2x}\, dx$$

$$= x - 6 \cdot \tfrac{1}{2} e^{2x} + C = \underline{\underline{x + 3e^{2x} + C}}$$

(2) Often you'll be asked to solve an Initial Value Problem
Find the solution satisfying $y(0) = 5$

$$\underset{\overset{\|}{5}}{y(0)} = 0 + 3e^{2 \cdot 0} + C = 3 + C \implies C = 2$$

$$\boxed{y = x + 3e^{2x} + 2}$$

Ex: $y' = y \cdot x$ this is a "separable" D.E.

(1) Find gen'l sol'n

$$\frac{dy}{dx} = y \cdot x \implies \frac{dy}{y} = x\, dx$$

Now integrate both sides (wrt y on left, x on rt.)

$$\int \frac{dy}{y} = \int x\, dx$$

$$\ln|y| = \frac{x^2}{2} + C \quad \text{becomes an exponent}$$

$$\implies e^{\ln|y|} = e^{\frac{x^2}{2} + C}$$

$$y \overset{e^w > 0}{=} |y| = e^{\frac{x^2}{2} + C} = e^{\frac{x^2}{2}} \cdot e^{C} = A \cdot e^{\frac{x^2}{2}} \qquad \text{constant, } A = e^C$$

Separable D.E.
$\frac{dy}{dx} = f(x) \cdot g(y)$
$\frac{1}{g(y)} \cdot dy = f(x)\, dx$
Separate x & y variables w/ $*$, $\div$
$\underline{\text{Not}}$ separable $\frac{dy}{dx} = y + x$

$$e^{A+B} = e^A \cdot e^B$$

$$\boxed{y = Ae^{\frac{x^2}{2}}} \quad \text{general sol'n}$$

(2) Solve the I.V.P. $y' = y \cdot x$, $y(4) = 5$.

use gen'l sol: $y = Ae^{x^2/2}$

$$\underset{\overset{\|}{5}}{y(4)} = Ae^{\frac{4^2}{2}} = Ae^8 \quad , \text{ solve for } A: \quad A = \frac{5}{e^8} \approx .0017$$

$$\boxed{y = 0.0017 \cdot e^{x^2/2}}$$

Find the "particular sol'n" to $\frac{dy}{dx} = 6xy^2$ w/ $y(0) = -1$

see: separable

particular sol'n

$$y = \frac{-1}{3x^2 - 1}$$

① $\frac{1}{y^2}\,dy = 6x\,dx$

② $\int \frac{1}{y^2}\,dy = \int 6x\,dx$

③ $-y^{-1} = \frac{6x^2}{2} + C$

④ solve for y: $\frac{-1}{y} = 3x^2 + C \Rightarrow y = \frac{-1}{3x^2 + C}$ gen'l sol'n

⑤ sub $1 = y(0) = \frac{-1}{3x^2 + C}$ solve for C : $3x^2 + C = -1$ $C = -1$