Wk 12 Wed

11-1 Common Parametric Equations

unit circle

$x^2+y^2=1$

$x = x(t) = \cos t$

$y = y(t) = \sin t$

circle w/ radius r

$x(t) = r\cos t$

$y(t) = r\sin t$

start $x^2+y^2=r^2$

$\Rightarrow \dfrac{x^2}{r^2} + \dfrac{y^2}{r^2} = 1$

$\Rightarrow \left(\dfrac{x}{r}\right)^2 + \left(\dfrac{y}{r}\right)^2 = 1$

$\cos^2 t + \sin^2 t = 1$

circle w/ radius $= r$, center $= (h,k)$

$x(t) = h + r\cos t$

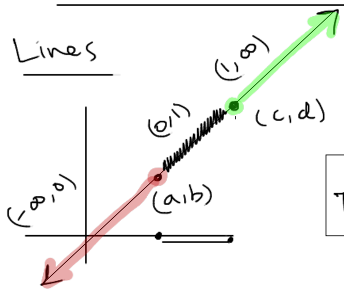
$y(t) = k + r\sin t$

ellipse @ $(0,0)$

$x(t) = a\cos t$

$y(t) = b\sin t$

Lines



$x(t) = a + t(c-a)$

$y(t) = b + t(d-b)$

$x(0) = a$

$x(1) = c$

$y(0) = b$

$y(1) = d$

$\boxed{\text{Thru two points}}$

$\boxed{\text{Line thru point w/ given slope}}$

given

Point: (a,b)

Slope: m

$\boxed{\text{Note: slope} = \dfrac{y'}{x'}}$

$x(t) = a + t$

$y(t) = b + mt$

In general,

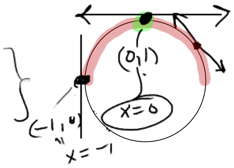
$\left.\begin{aligned} x(t) &= a + rt \\ y(t) &= b + st \end{aligned}\right\}$ Line thru (a,b) w/ slope: $m = \dfrac{s}{r}$

Parametric Functions describe curves, and these curves have slopes of tangent lines

Ex

$x(t) = \cos(t)$

$y(t) = \sin(t)$



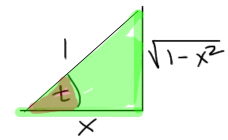
what is the slope of tangent line?

① go from parametric to $y = f(x)$ by "eliminating the parameter": solving $x = \cos t$ for t, then plug in to y.

$t = \cos^{-1}(x)$

$y = \sin(\cos^{-1}(x))$

opp / hyp   angle



$y = \sqrt{1-x^2}$

② $\frac{dy}{dx} = \frac{1}{2}(1-x^2)^{-1/2}(-2x) = \boxed{\frac{-x}{\sqrt{1-x^2}}}$

slope of tangent line

x = -1

x = 0

$\boxed{\text{ALT. METHOD OF FINDING } \frac{dy}{dx}}$

use "chain rule"

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'}{x'}$$

$$\frac{dy}{dx} = \frac{y'}{x'} = \frac{\cos t}{-\sin t} = -\cot(t)$$

$$= -\cot(\cos^{-1}(x))$$

$$= -\frac{\text{adj}}{\text{opp}} = \frac{-x}{\sqrt{1-x^2}}$$

Ex $x = s^3 \quad y = 5s^6 - s^{-3}$ Find $\frac{dy}{dx}$ slope of tangent line

$$\frac{dy}{dx} = \frac{y'}{x'} = \frac{30s^5 + 3s^{-4}}{3s^2} = 10s^3 + s^{-6}$$

$\underbrace{\text{get } x \text{ instead of } s,}_{\text{use given: } x = s^3}$

$x^{1/3} = s$

$$\frac{dy}{dx} = 10\left(x^{1/3}\right)^3 + \left(x^{1/3}\right)^{-6} = 10x + x^{-2} = 10x + \frac{1}{x^2}$$

slope @ ∞: $\lim_{x \to \infty} \frac{dy}{dx} = 10 \cdot \infty + \frac{1}{\infty^2} = \infty$