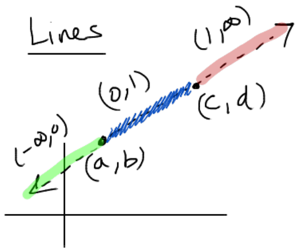
Wk 12 Wed

11-1 Common Parameterizations

Unit Circle	Circle w/ Radius r, center $= (0,0)$	circle, radius $= r$, center $= (h,k)$	ellipse: $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ @ $(0,0)$
$x = x(t) = \cos(t)$	$x(t) = r\cos(t)$	$x(t) = h + r\cos t$	$x(t) = a\cos(t)$
$y = y(t) = \sin(t)$	$y(t) = r\sin(t)$	$y(t) = k + r\cos t$	$y(t) = b\sin(t)$

Lines



this Parametrization will be

$$\begin{cases} (a,b) & t = 0 \\ (c,d) & t = 1 \\ \text{between them} & t \in (0,1) \end{cases}$$

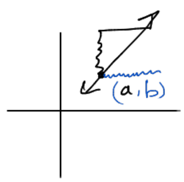
$x(t) = a + t(c-a)$

$y(t) = b + t(d-a)$

Line thru (a,b) w/ slope $\underline{\underline{m}} = \frac{\text{rise}}{\text{run}} = \frac{m}{1}$

$x(t) = a + t$

$y(t) = b + mt$



In general

Line thru a,b w/ slope $\frac{s}{r}$

$x = a + rt$

$y = b + st$

(Note: slope $= \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y'}{x'} = \frac{s}{r}$

$\frac{dy}{dt} = s$

$\frac{dx}{dt} = r$

Sometimes (when a coord. function is $\underline{\text{invertible}}$)
you can create a function $y = f(x)$ from parametric equations

$x = x(t)$
$y = y(t)$

$\underline{\text{Ex}}$ $x = 1 + 3t$, $y = -3 + 8t$

Determine $y = f(x)$. Determine y as a function of x - not t

① eliminate the parameter: start w/ x & $\underline{\text{solve for } t}$

$x = 1 + 3t$

$\Rightarrow \quad x - 1 = 3t$

$\frac{x-1}{3} = t$

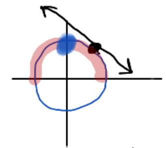
② sub this value of t into y:

$y = -3 + 8t$
$= -3 + 8\left(\frac{x-1}{3}\right)$
$= -3 + \frac{8x-8}{3} = 2.\bar{6}x - 5.\bar{6}$

$$\boxed{y \approx 2.6x - 5.6}$$

Recall:

$x(t) = \cos(t)$
$y(t) = \sin(t)$ determine a graph



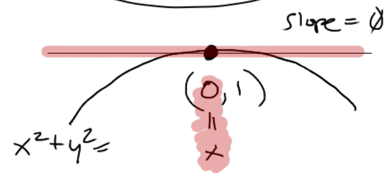
this graph has a slope of tangent line

typically:

$y = \sqrt{1-x^2}$

$y' = \frac{1}{2}(1-x^2)^{-1/2}(-2x)$

$\boxed{\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}}$



To determine $\frac{dy}{dx}$ when given parametric eqn's:

(1) use chain rule: $\frac{dy}{dx} = \frac{\frac{dy}{\cancel{dt}}}{\frac{dx}{\cancel{dt}}}$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

i.e., $\frac{dy}{dx} = \frac{y'}{x'} = \frac{\cos t}{-\sin t} = -\cot(t)$

(2) Now get t out & x in

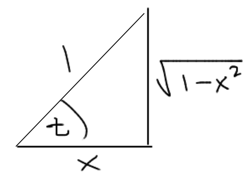
since $x = \cos t$

$\cos^{-1}(x) = t$

(3) $\frac{dy}{dx} = -\cot(\cos^{-1}(x))$

$= -\frac{\text{adj}}{\text{opp}}$

$= \frac{-x}{\sqrt{1-x^2}}$



Ex

$x = x(s) = s^3$

$y(s) = 5s^6 + s^{-3}$

$$\frac{dy}{dx} = \frac{y'}{x'} = \frac{30s^5 - 3s^{-4}}{3s^2} = 10s^3 - s^{-6} = 10(x^{1/3})^3 - (x^{1/3})^{-6}$$

$$= 10x - x^{-2} = \frac{dy}{dx}$$

s out, x in

$x = s^3$

$s = x^{1/3}$