163 Take Home Exam - Arc Length

1. Use the formula for arc length and the equation $x^2 + y^2 = R^2$ to derive the formula for the circumference of a circle with radius R.

2. Sketch and find the arc length of the curve below.

$$x = \cos^3 t, \ y = \sin^3 t$$

3. Show that the exact length of the curve below is 2.

$$y = \sqrt{x - x^2} - \sin^{-1}\left(\sqrt{x}\right)$$

163 Take Exam 3 - Surface of a Revolution

1. Use the formula for surface area and the equation $x^2 + y^2 = R^2$ to derive the formula for the surface area of a sphere with radius R.

2. Find the surface area of the solid obtained by revolving the curve $y = \sin^3 t$, $x = \cos^3 t$ about the x-axis.

3. Gabriel's horn is formed by taking the graph of $y = \frac{1}{x}$ for $x \geq 1$ and rotationg it about the x-axis.

    (a) Find the volume of Gabriel's Horn.

    (b) Set up an integral that represents the surface area of Gabriel's Horn.

    (c) Show that the surface area of Gabriel's Horn is infinite.

$$Hint: \quad \sqrt{1 + \frac{1}{x^4}} > 1 \text{ for all } x \geq 1$$

163 Take Home Exam - Differential Equations

Solve the following initial value problems. The first two are separable differential equations, and the last two are first order linear differential equations.

1. $\dfrac{dy}{dx} = \dfrac{\sin x}{\cos y}$; $y(0) = \pi$

2. $\dfrac{dy}{dx} = \dfrac{(y^2 + 1)\sec^2 x}{y}$; $y(0) = 0$

3. $y' - 4y = 8x, y(0) = 5$

4. $y' - \dfrac{3}{x}y = 4, y(1) = 7$

163 Take Home Exam - Centers of Mass

1. Find the centroid of the region bounded by $y = 4 - x^2$ and the x-axis.