Fri wk 13

Exam 4 Take Home : Po<u>st</u>ed

Today: 11-4 Homework

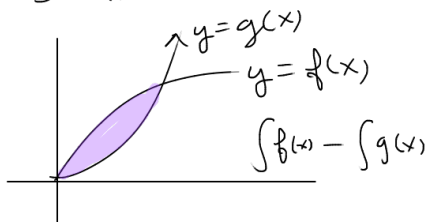Mon - Wed - Fri : Review
(Study Guide

Final: tuesday
       Monday
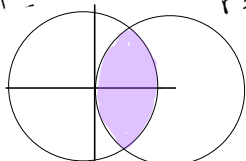
Ex Areas b/w curves _____
Similar to rectangular coords: Subtraction is key
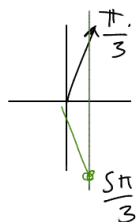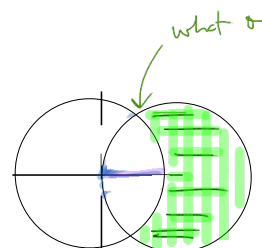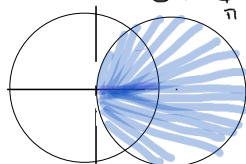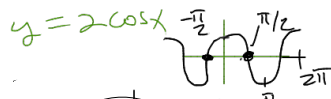 even in Polar setting!
$y = g(x)$
$y = f(x)$
$\int f(x) - \int g(x)$

Ex $r = 1$. $r = 2\cos(\theta)$ → $y = 2\cos x$

what is the shaded area

$r = 1 = 2\cos\theta$

$\frac{1}{2} = \cos\theta$

$\cos^{-1}\left(\frac{1}{2}\right) = \theta$

to find the green area below
subtract A from B
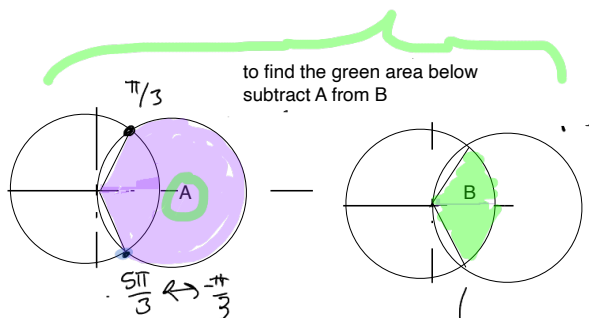
$\frac{1}{2}\int_{-\pi/3}^{\pi/3}(2\cos\theta)^2 d\theta - \frac{1}{2}\int_{-\pi/3}^{\pi/3}(1)^2 d\theta$

In conclusion:

$\frac{1}{2}\int_{-\pi/2}^{\pi/2}(2\cos\theta)^2 d\theta - \frac{1}{2}\left[\int_{-\pi/3}^{\pi/3}(2\cos\theta)^2 d\theta - \int_{-\pi/3}^{\pi/3}(1)^2 d\theta\right]$

To compute
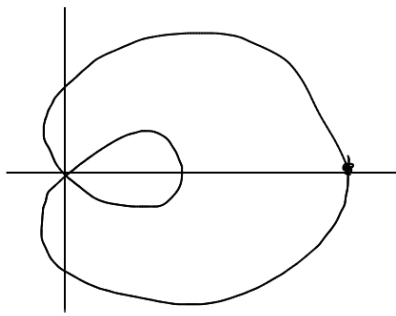integral use:

$\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$

$$r = 4\cos\theta + 1$$

$\downarrow$ rect. coords

$$y = 4\cos x + 1$$

$y = 0$

$$4\cos x + 1 = 0$$
$$x = \cos^{-1}\left(-\tfrac{1}{4}\right)$$

$$\cos^{-1}\left(-\tfrac{1}{4}\right)$$

$$2\pi - \cos^{-1}\left(-\tfrac{1}{4}\right)$$

TOP HALF OF OUTER LOOP

$$\frac{1}{2}\int_0^{\cos^{-1}\left(-\tfrac{1}{4}\right)} (4\cos\theta + 1)^2 \, d\theta$$

TOTAL ENCLOSED AREA

$$\int_0^{\cos^{-1}\left(-\tfrac{1}{4}\right)} (4\cos\theta + 1)^2 \, d\theta$$

Inner Circle

$$\frac{1}{2}\int_{\cos^{-1}\left(-\tfrac{1}{4}\right)}^{2\pi - \cos^{-1}\left(-\tfrac{1}{4}\right)} (4\cos\theta + 1)^2 \, d\theta$$

Shaded Area = Difference $-\nearrow$