Fri. wk 13

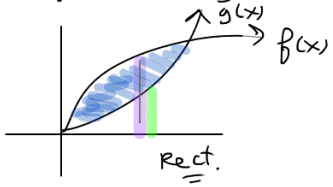
Exam 4 - Posted, due Thurs. (Presentations)

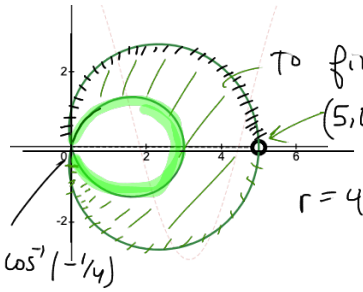
Mon. Wed Fri: review for final

Exercise From H/W:

Area b/w curves:

Similar to rectangular coords: (subtract), same for Polar Coords.

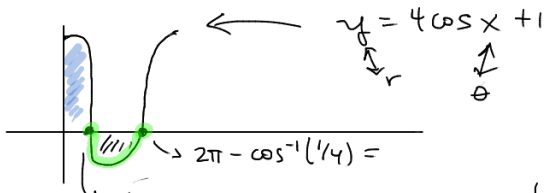




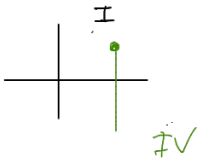
To find the shaded area: subtract

$r = 4\cos\theta + 1$

To find parameters for each branch plot in rect. coords (or make table)



$4\cos x + 1 = 0 \Rightarrow \cos x = -\frac{1}{4}$, $x = \cos^{-1}(-1/4)$



Top Half of Large Section:

$$\frac{1}{2}\int_0^{\cos^{-1}(-1/4)} r^2\, d\theta$$

$$\frac{1}{2}\int_0^{\cos^{-1}(-1/4)} (4\cos\theta + 1)^2\, d\theta$$

Total Enclosed Area:

$$\int_0^{\cos^{-1}(-1/4)} (4\cos\theta + 1)^2\, d\theta$$

Inner Small Loop:

$$\frac{1}{2}\int_{\cos^{-1}(-1/4)}^{2\pi - \cos^{-1}(-1/4)} (4\cos\theta + 1)^2\, d\theta$$

Shaded Area:

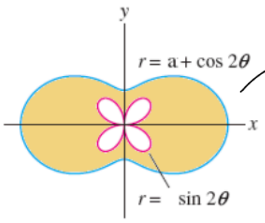
Subtract. Total − Inner.

use: $(4\cos\theta + 1)^2 = 16\cos^2\theta + 8\cos\theta + 1$

$\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$

Question 4 of 16

Find the area A between the two curves for $a = 21$.



use symmetry: work in quadrant I, multiply by 4.

Quad: I $\theta = 0$, $\theta = \frac{\pi}{2}$

$r = 21 + \cos 2\theta$ outer

$$\frac{1}{2}\int_0^{\pi/2} (21 + \cos 2\theta)^2 \, d\theta - \frac{1}{2}\int_0^{\pi/2} (\sin 2\theta)^2 \, d\theta$$

same bounds

$$= \frac{1}{2}\int_0^{\pi/2} (21 + \cos 2\theta)^2 - \sin^2(2\theta) \, d\theta$$

$\cos^2 + \sin^2 = 1$

$$= \frac{1}{2}\int_0^{\pi/2} 21^2 + 42\cos 2\theta + \cos^2(2\theta) - \underbrace{\sin^2(2\theta)}_{-(1-\cos^2(2\theta))} \, d\theta = \frac{1}{2}\int_0^{\pi/2} 21^2 + 42\cos 2\theta + \underbrace{2\cos^2(2\theta)}_{\text{half-angle formula}} - 1 \, d\theta$$

$-1 + \cos^2(2\theta)$

what is the set of parameters theta for the graph below?

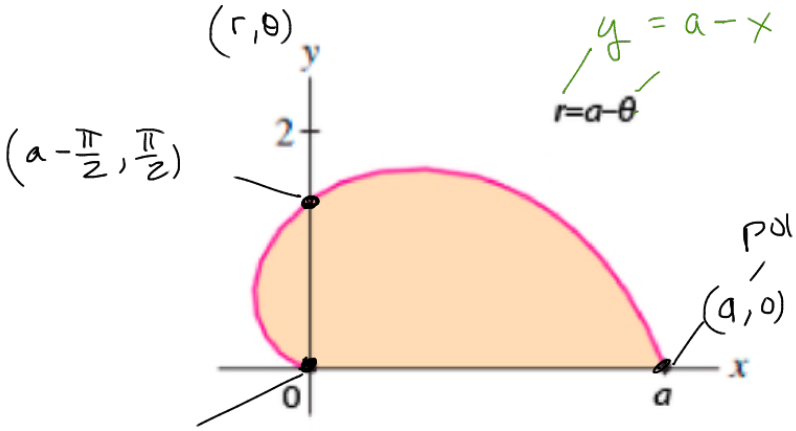
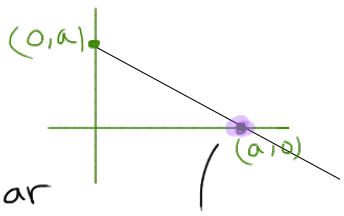


table
rect. coords

$y = a - x$



y is playing role of radius ⇒

when $y = 0$ then $r = 0$ so the graph is @ (0,0)

$[0, a]$