

Week 13 - Monday

Last new topic: Differential Equations

this week: (Wed.) get your take home exam,

Calculus: mathematics of change

derivatives express change / integrals express accumulation of change

Common use of Calculus: set up/solve equations involving an unknown function & its derivatives — a differential equation

A solution to a differential equation (D.E.) is a function, that when differentiated/substituted solves the D.E.

Ex	D.E.	Solutions
1.	$y' = 2x$	$y = x^2$ or $y = x^2 + 7$ , etc.,
2.	$y' + 3y = 6x + 11$ <u>first-degree</u>	$y = e^{-3x} + 2x + 3$ (b/c: $d/dx$ ) = $y' = -3e^{-3x} + 2$ ... substitute into LHS LHS of D.E. $-3e^{-3x} + 2 + 3(e^{-3x} + 2x + 3) = 2 + 6x + 9 = 6x + 11 = \underline{\underline{RHS}}$
3.	$y'' - 3y' + 2y = 24e^{-x}$ <u>second degree D.E.</u>	$y = 3e^{3x} - 4e^{2x} + 2e^{-2x}$

General & Particular Solutions

↓  
all possible solutions

↓  
Just one solution (one function) that solves the D.E.

ex: For  $y' = 2x$   
the gen'l sol'n is  $y = x^2 + C$  where  $C \in \mathbb{R}$  is constant

Methods of solving D.E.'s

- there are many: Math 365 = (Diff Eq's)

- We'll study separable differential equations:

look like:  $y' = f(x) \cdot g(y)$

solve by:  $\frac{y'}{g(y)} = f(x)$

dividing by  $g(y)$

integrating both sides:

Ex  $y' = (x^2 - 4)(3y + 2)$  (separable) 

(5-step):

1. Look for constant sol'n: Find a  $y$ -value that makes  $g(y) = 0$ . (solve  $3y + 2 = 0$ )

$y = -\frac{2}{3}$  this a sol'n ✓

$(-\frac{2}{3})' = (x^2 - 4)(\underbrace{3(-\frac{2}{3} + 2)}_{=0})$

2. Separate / Integrate

$y' = \frac{dy}{dx}$  ratio of differentials  $\frac{dy}{3y+2} = x^2 - 4$   
 ↓ integrate both wrt  $x$

$\int \frac{dy}{3y+2} dx = \int x^2 - 4 dx$

\* Frequently absorb all constants into one

$\int \frac{dy}{3y+2} = \int x^2 - 4 dx$

(rename C)

$\frac{1}{3} \ln|3y+2| + D = \frac{x^3}{3} - 4x + C$   $C_1 = C - D$

$\ln|3y+2| = x^3 - 12x + C_2$   $C_2 = 3C_1$

3. Solve for  $y$

$|3y+2| = e^{x^3 - 12x + C_3} = e^{x^3 - 12x} \cdot e^{C_3}$   $C_4 = e^{C_3} > 0$

$|3y+2| = C_4 e^{x^3 - 12x}$

$3y+2 = \pm C_4 e^{x^3 - 12x}$   $C_5 = \pm C_4$

$3y+2 = C_5 e^{x^3 - 12x}$

$y = \frac{C_5 e^{x^3 - 12x} - 2}{3}$

general sol'n.

4. Find the particular sol'n if appropriate

RHS = 1 =  $\frac{C_5 \cdot e^{-27+36} - 2}{3}$  &  $C_5 = \frac{3+2}{e^9} = \frac{5}{e^9}$

if  $y = 1$  when  $x = -3$   
 (determines some particular sol'n.)

$y = \frac{(5/e^9) e^{x^3 - 12x} - 2}{3}$