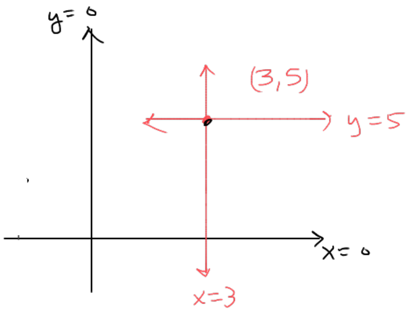
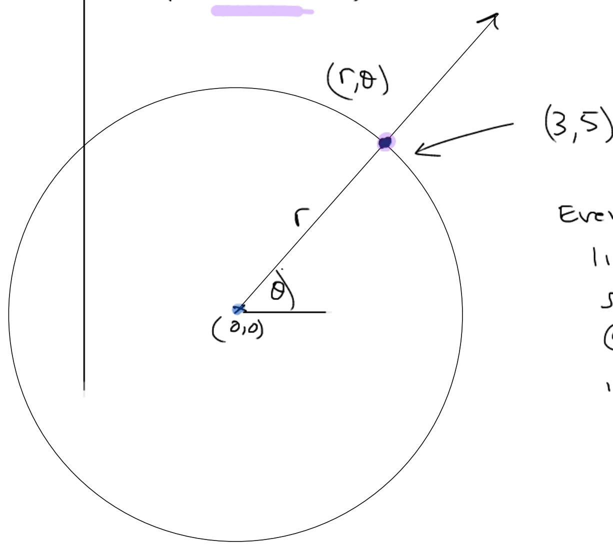
MA163 wk 13 - Mon

Polar Coordinates (11-3)

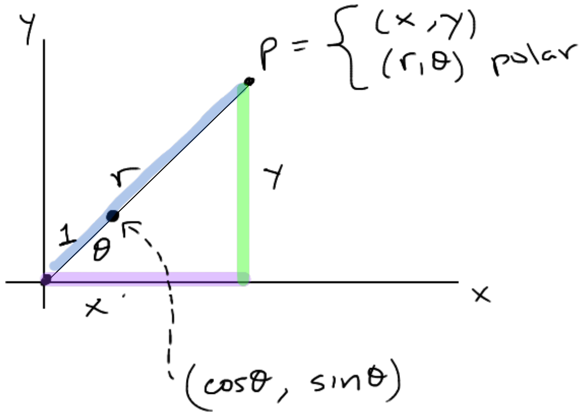
Usually, Rectangular (Cartesian)



Polar Coords



Every point lies on some ray @ angle θ intersected with circle, radius = r



polar to rectangular

$x = r\cos\theta$

$y = r\sin\theta$

rectangular to polar

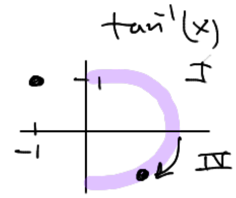
$r = \sqrt{x^2+y^2}$

$\tan\theta = \dfrac{y}{x} \quad (x \neq 0)$

(often: $\tan^{-1}\left(\frac{y}{x}\right) = \theta$

Ex: polar $\left(3, \frac{\pi}{4}\right) \rightsquigarrow$ $x = 3 \cdot \cos\frac{\pi}{4} = \dfrac{3\sqrt{2}}{2}$ $\rightsquigarrow$ rectangular $\left(\dfrac{3\sqrt{2}}{2}, \dfrac{3\sqrt{2}}{2}\right)$

$y = 3\sin\frac{\pi}{4} = \dfrac{3\sqrt{2}}{2}$



Ex. Rectangular $(-1, 1) \rightsquigarrow r = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$

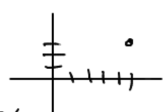
$\tan\theta = \dfrac{-1}{1} = -1 \Rightarrow \theta = \tan^{-1}(-1) = -\dfrac{\pi}{4}$

since $x < -1 \Rightarrow \theta = \tan^{-1}\left(\frac{y}{x}\right) + \pi = -\dfrac{\pi}{4} + \pi = \dfrac{3\pi}{4}$ $\boxed{\left(\sqrt{2}, \frac{3\pi}{4}\right)}$

Reason: Convention: angle is in $[0, 2\pi)$

Note: Rectangular Coords:

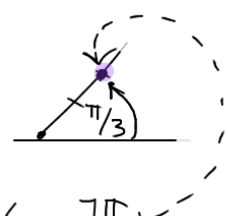
$(5, 3) \rightarrow$



gives a unique point

Polar Coords

$\left(2, \frac{\pi}{3}\right) \longrightarrow$

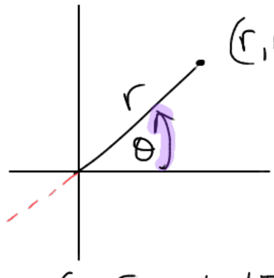


$\left(2, \frac{\pi}{3} + 2\pi\right) = \left(2, \frac{7\pi}{3}\right)$

Note: $-r$ is allowed. It just means "the opposite point"

$(r,\theta) \rightsquigarrow$

$(-r,\theta)$



if $r > 0$ then
$$\theta = \begin{cases} \tan^{-1}(y/x) & x > 0 \\ \tan^{-1}(y/x) + \pi & x < 0 \\ \pm\frac{\pi}{2} & x = 0 \end{cases}$$

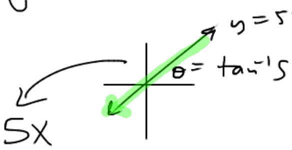
MATLAB, R, Python

Coding: atan2 implements this, so that θ matches the coord. of (x,y)

Lines

Rectangular

$y = m x + b$



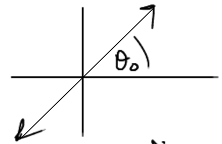
Ex. $y = 5x$

$5 = \tan\theta \Rightarrow \underbrace{\theta = \tan^{-1}(5)}$

determines a line in Polar coords.

Polar

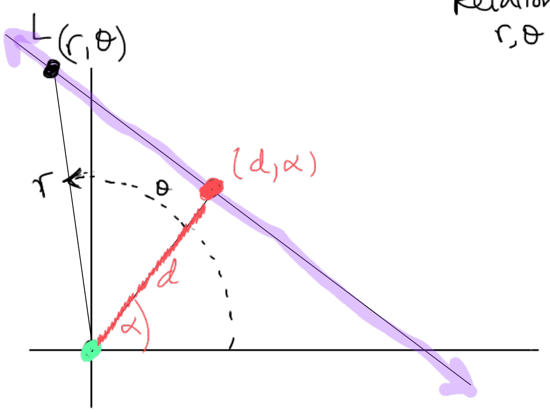
(Line thru origin:)

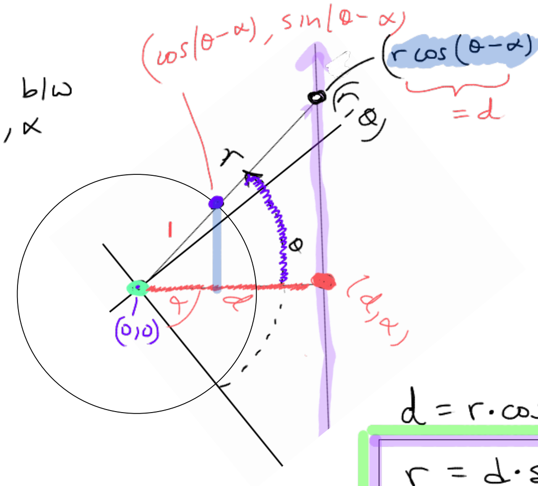


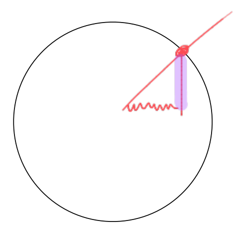
$\theta = \theta_0$

Line not thru origin

Relationship b/w r, θ, d, α







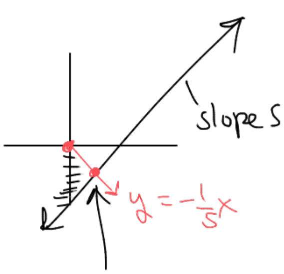
$d = r \cdot \cos(\theta - \alpha)$

$$\boxed{r = d \cdot \sec(\theta - \alpha)}$$

Ex. Determine polar eqn of line thru $(2,3)$ with slope 5, rectangular

$y - 3 = 5(x-2)$

$$\boxed{y = 5x - 7}$$



① Determine point on line closest to origin

$\Rightarrow m_\perp = -\frac{1}{5}$ (slope)

$\Rightarrow$ point $= (0,0)$

② combine eqns

$y = -\frac{1}{5}x = 5x - 7$

$-x = 25x - 35$

$35 = 26x$

$\frac{35}{26} = x$

$\left(\frac{35}{26}, 5\left(\frac{35}{26}\right) - 7\right)$

$\parallel$ x $\parallel$ y

$d = \sqrt{x^2 + y^2}$

$\alpha = \tan^{-1}(y/x)$

$= r = d\sec(\theta - \alpha)$

eg, $\boxed{r = 1.4 \cdot \sec(\theta - 0.2)}$