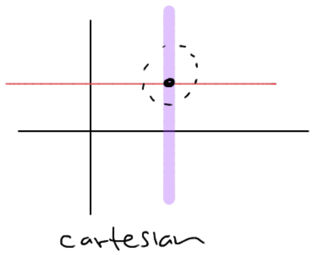
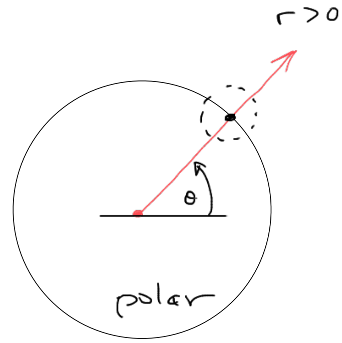
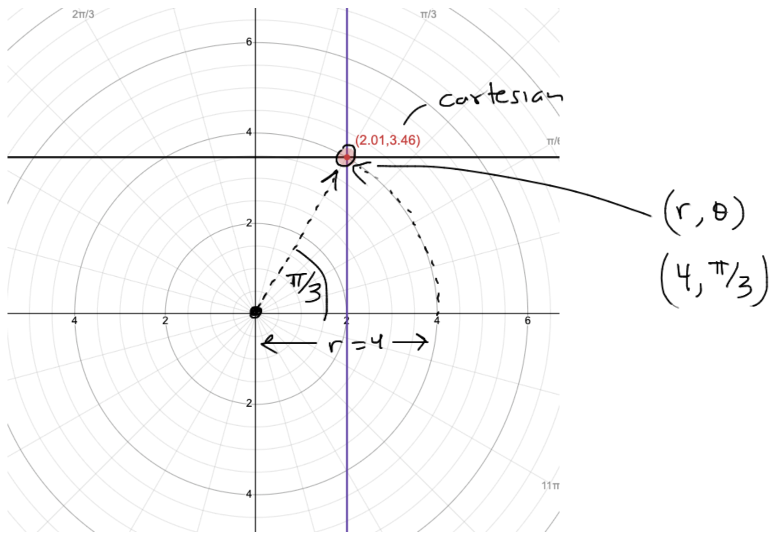
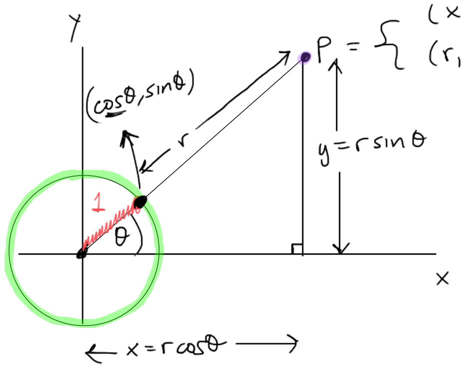
Wk 13 Mon

Last Topic: Polar Coordinates

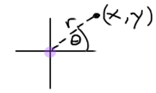








$P = \begin{cases} (x,y) & \text{rectangular (Cartesian)} \\ (r,\theta) & \end{cases}$

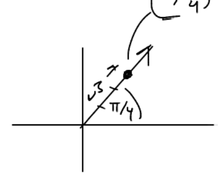


$\tan\theta = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y}{x}$

Polar to Rectangular	Rectangular to Polar
$x = r\cos\theta$	$r = \sqrt{x^2+y^2}$
$y = r\sin\theta$	$\tan\theta = \frac{y}{x}$ $(x \neq 0)$ (often: $\theta = \tan^{-1}(\frac{y}{x})$)

polar rectangular

$(3, \frac{\pi}{4}) = (\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2})$



From Polar to Rectangular

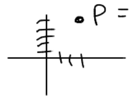
Ex. $r = 3,\ \theta = \frac{\pi}{4}$ determines a point $(3, \frac{\pi}{4})$

$x = 3\cos\frac{\pi}{4} = \frac{3\sqrt{2}}{2}$

$y = 3\sin\frac{\pi}{4} = \frac{3\sqrt{2}}{2}$

Ex From Rect. to Polar

$x = 3,\ y = 5$



$\bullet P = \begin{cases} (3,5) & \text{Rect.} \\ (5.9, 1.03) & \text{Polar} \end{cases}$

$r = \sqrt{3^2+5^2} = \sqrt{34} \approx 5.9$

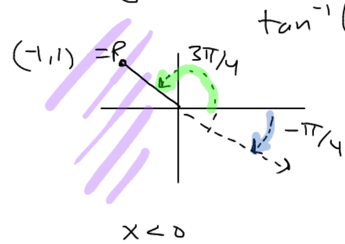
$\tan\theta = \frac{5}{3}$, $\theta = \tan^{-1}(\frac{5}{3}) \approx 1.03$

want: in $(0, 2\pi)$

If $r > 0$, the "θ"-coordinate of $P = (x, y)$
↑rectangular

$$\theta = \begin{cases} \tan^{-1}\left(\frac{y}{x}\right) & x > 0 \\ \tan^{-1}\left(\frac{y}{x}\right) + \pi & x < 0 \\ \pm\frac{\pi}{2} & x = 0 \end{cases}$$

$\tan^{-1}\left(\frac{-1}{1}\right) = \tan^{-1}(-1)$
$= -\frac{\pi}{4}$

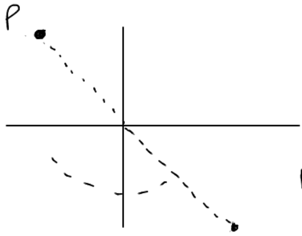


$-\frac{\pi}{4} + \pi = \frac{3\pi}{4}$

Note: $r < 0$ is also used

$\sqrt{2} = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$

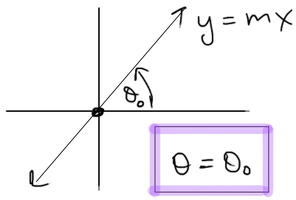
$(-1,1) = P$
$\shortparallel$
$\left(\sqrt{2}, \frac{3\pi}{4}\right)$
$\shortparallel$
$\left(-\sqrt{2}, \frac{7\pi}{4}\right)$



$r = -\sqrt{2}$ ($\Rightarrow$ opposite direction)

Lines:

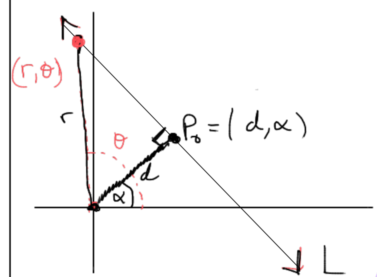
Rectangular

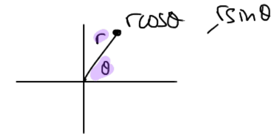


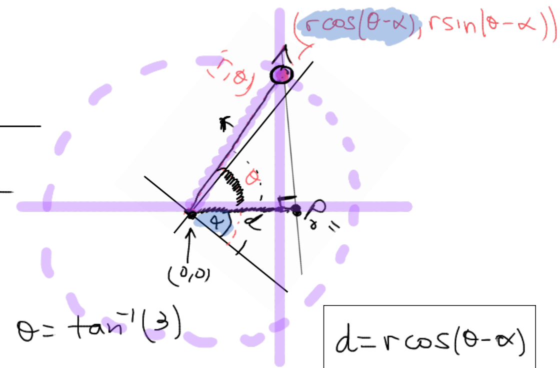
$$\boxed{\theta = \theta_0}$$

Polar coord eq'n of a line thru origin

Lines not thru origin







$$\boxed{d = r\cos(\theta - \alpha)}$$

solve for r

$$\boxed{r = d\sec(\theta - \alpha)}$$

Ex $\boxed{y = 3x}$, in polar coords, since $\tan\theta = 3$, $\theta = \tan^{-1}(3)$

$$\boxed{\theta = \tan^{-1}(3)}$$