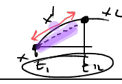
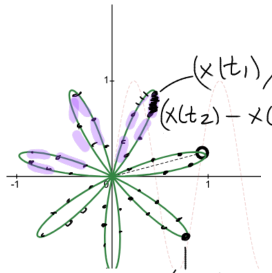
thur wk 13 section 11-4

ARC LENGTH :

Recall:



$$\text{Approx Length} = \sum_{i=1}^{N} \sqrt{\underbrace{(x(t_{i+1}) - x(t_i))^2 + (y(t_{i+1}) - y(t_i))^2}_{\text{mean value thm:}}}$$



$(x(t_1), y(t_1))$

$(x(t_2) - x(t_2))$

(r, θ)

$\|$

$(f(\theta), \theta)$

$$= \sum_{i=1}^{N} \sqrt{(x'(t_i^*) \cdot \Delta t)^2 + (y'(t_i^*) \cdot \Delta t)^2}$$

$$= \sum_{i=1}^{N} \sqrt{(x'(t))^2 + (y'(t))^2}\, \Delta t$$

Exact length $\lim_{N \to \infty}$ $= \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$ ← ARC length for parametric curve

To make it Polar:

$x = r\cos\theta = f(\theta) \cdot \cos\theta$

$y = r\sin\theta = f(\theta)\sin\theta$

$(x')^2 = (f'(\theta)\cos\theta - f(\theta)\sin\theta)^2 = (f'(\theta))^2 \cdot \cos^2\theta - 2f'(\theta)\cos\theta f(\theta)\sin\theta + f^2(\theta)\sin^2\theta$

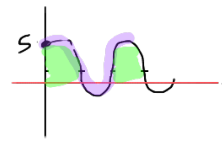
$+ \; (y')^2 = (f'(\theta)\sin\theta + f(\theta)\cos\theta)^2 = (f'(\theta))^2\sin^2\theta + 2f'(\theta)\sin\theta f(\theta)\cos\theta + f^2(\theta)\cos^2\theta$

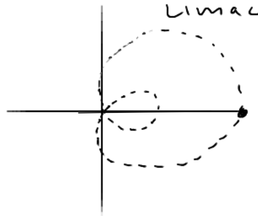
$(x'(\theta))^2 + (y'(\theta))^2 = (f'(\theta))^2 + f^2(\theta)$

$$\boxed{\text{Polar Arc length} = \int_\alpha^\beta \sqrt{f^2(\theta) + f'(\theta)^2}\, d\theta}$$

Ex: $r = \underbrace{1+4\cos\theta}_{f(\theta)}$, Find total length.

r	5
θ	0

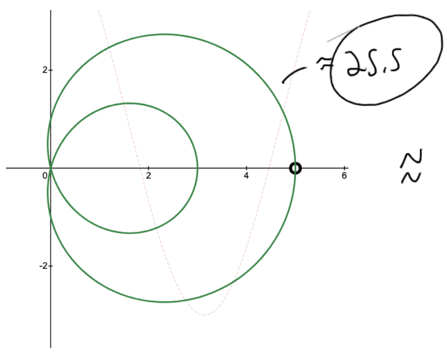




$$s = \int_0^{2\pi} \sqrt{(1+4\cos\theta)^2 + (-4\sin\theta)^2}\, d\theta$$

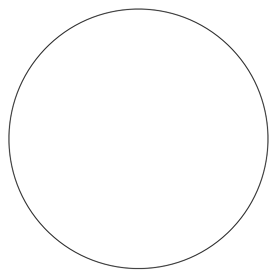
$$= \int_0^{2\pi} \sqrt{1 + 8\cos\theta + 16\cos^2\theta + 16\sin^2\theta}\, d\theta = \int_0^{2\pi} \sqrt{17 + 8\cos\theta}\, d\theta$$

=



≈ 25.5

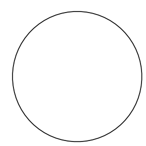
$\approx$



$r = 2.5$

$C = 2\pi r$

$+$



$r = 1.5$

$\approx 2\pi(2.5) + 2\pi(1.5)$

$2\pi(4) = 8\pi$

≈ 25.1