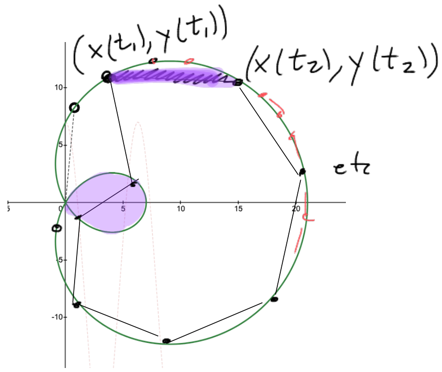
Thurs wk 13

Section 11-4

Arc Length in Polar Coords



Approx length: $\displaystyle\sum_{i=1}^{N} \sqrt{[x(t_{i+1}) - x(t_i)]^2 + [y(t_{i+1}) - y(t_i)]^2}$ (N → # of segments)

Exact length: $\displaystyle\lim_{N\to\infty} \sum_{i=1}^{N} \sqrt{\underbrace{[x(t_{i+1}) - x(t_i)]^2}_{} + \underbrace{[y(t_{i+1}) - y(t_i)]^2}_{}}$

MEAN VALUE THM

$$= \lim_{N\to\infty} \sum^{N} \sqrt{[x'(t_i^*)\Delta t_i]^2 + [y'(t_i^*)\Delta t_i]^2}$$

$$\boxed{s = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} \cdot dt}$$

Arc length of Parametric Curve

To get this into Polar Coords:

$r = f(\theta)$

$x = r\cos\theta$, $y = r\sin\theta$

$= f(\theta)\cdot\cos\theta$, $= f(\theta)\sin\theta$

So $x' = f'(\theta)\cdot\cos\theta - f(\theta)\sin\theta$, $y' = f'(\theta)\sin\theta + f(\theta)\cos\theta$

$$(x')^2 = (f'(\theta))^2\cos^2\theta \quad \boxed{-2f'(\theta)f(\theta)\cos\theta\sin\theta} \quad + f(\theta)^2\sin^2\theta$$

$$+ \quad (y')^2 = (f'(\theta))^2\sin^2\theta \quad \boxed{+ 2f'(\theta)f(\theta)\sin\theta\cos\theta} \quad + (f(\theta))^2\cos^2\theta$$

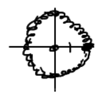
$$(x')^2 + (y')^2 = (f'(\theta))^2 + (f(\theta))^2$$

POLAR ARC LENGTH

$$\boxed{S_P = \int_\alpha^\beta \sqrt{f(\theta)^2 + f'(\theta)^2}\, d\theta}$$

Ex $r = 2 \Rightarrow$

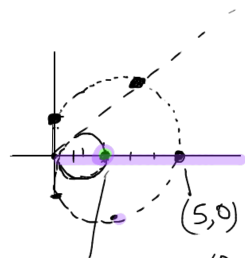
$f(\theta) = 2$



$C = 2\pi r$
$= 2\pi \cdot 2 = 4\pi$

$$\int_0^{2\pi} \sqrt{(\underbrace{f(\theta)}_{= 2})^2 + (\underbrace{f'(\theta)}_{= 0})^2}\, d\theta = \int_0^{2\pi} \sqrt{2^2 + 0^2}\, d\theta = 2\int_0^{2\pi} d\theta = 2\theta \Big|_0^{2\pi} = 4\pi$$

Ex $r = 1 + 4\cos(\theta)$
$= f(\theta)$



$(r, \theta) = (3, 0)$
$(-r, \theta + \pi) = (-3, \pi)$

$r = 1 + 4\cos(\theta)$

r	5	≈ 4	1	1	-3	
θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	π	

$1 + 4\cos\frac{\pi}{4} = 4\frac{\sqrt{2}}{2} + 1 = \underbrace{2\sqrt{2}}_{4} + 1$

$1 - 4 = -3$

$$\text{Arc length} = \int_0^{2\pi} \sqrt{\underbrace{(1 + 4\cos\theta)^2}_{} + (-4\sin\theta)^2}\, d\theta = \int_0^{2\pi} \underbrace{\sqrt{17 + 8\cos\theta}}_{}\, d\theta$$

$1 + 8\cos\theta + 16\cos^2\theta + 16\sin^2\theta$

use: $\cos^2\frac{\theta}{2} = \frac{1}{2}\cdot(1 + \cos\theta) \Rightarrow 2\cos^2\frac{\theta}{2} - 1 = \cos\theta$

$= \int_0^{2\pi} \sqrt{17 + 16\cos^2\frac{\theta}{2} - 8}$

$= \int_0^{2\pi} \sqrt{9 + 16\cos^2\frac{\theta}{2}}$