Wed. Week 13

Exam 3: Scores on Educat, get from me,

Exam 4: (Take Home) Notes page / Due classtime next Wednesday / work together ask me,

Final Exam: 12pm Wednesday

: 93 or better in est_grade $\Rightarrow$ you don't have to take the Final Exam

<u>Today</u>: Differential Equations (1st Order Linear)

We're learning two methods for solving D.E.'s

1. separable
2. $\underbrace{1^{st}\text{ order}}_{1^{st}\text{ derivative only}}$ linear : $(ax_1 + bx_2 = c)$ linear equation in variables x_1, x_2.

applies to : $a(x)y' + b(x)y = c(x)$.

1. get into std. form

$$y' + \frac{b(x)}{a(x)}y = \frac{c(x)}{a(x)}$$

2. rename!

$$\boxed{y' + p(x)\cdot y = q(x)}$$

$\left(p(x) = \frac{b(x)}{a(x)}, \text{etc}\right)$

3. Note: LHS looks (almost) like derivative w/ product rule

$$\frac{d}{dx}(f\cdot g) = f\cdot g' + f'g$$

4. Multiply by $u(x)$: (unknown)

$$u(x)y' + \underbrace{u(x)p(x)}_{\text{needs to be } u'(x)}y = u(x)q(x)$$

LHS $\underline{\underline{\text{is}}}$ $\frac{d}{dx}(y\cdot u(x))$ $\underline{\underline{\text{if}}}$ $u'(x) = u(x)p(x)$.
another D.E. (separable) $y' = (\text{one var})(\text{other var})$

5. $\frac{u'(x)}{u(x)} = p(x)$ $\xrightarrow[\int dx]{\text{From Monday}}$ $\int \frac{u'(x)}{u(x)}dx = \int p(x)dx$

$$\ln|u(x)| = \int p(x)dx + C$$

6. Raise $e^{\wedge}$: $|u(x)| = e^{\int p(x)dx + C} = e^{\int p(x)dx}\cdot e^{C} = C_1 e^{\int p(x)dx}$

drop abs value $e^x > 0$ | we need just one particular sol'n set $C_1 = 1$

$$\boxed{u(x) = e^{\int p(x)dx}}$$

integrating factor

7. So: given $y' + p(x)y = q(x)$

do: $e^{\int p(x)dx}(y' + p(x)y) = e^{\int p(x)dx}\cdot q(x)$

distribute

$$e^{\int p(x)dx}\cdot y' + e^{\int p(x)dx}\cdot p(x)\cdot y$$

derivative of product

$$\int \frac{d}{dx}\left(e^{\int p(x)dx}\cdot y\right)dx = \int e^{\int p(x)dx}\cdot q(x)\,dx$$

cancel

$$e^{\int p(x)}\cdot y = \int e^{\int p(x)dx}\cdot q(x)\,dx$$

$$\boxed{y = \frac{1}{e^{\int p(x)dx}}\int e^{\int p(x)dx}\cdot q(x)\,dx}$$

Ex

$$y' + 4y = 8$$

(0) Recognize: 1st Order Linear: $y' + p(x)y = q(x)$

(1) $p(x) = 4$, $\int 4\,dx = 4x$, $e^{4x} = u(x)$ ——— integrating factor

(2) $$e^{4x}(y' + 4y) = e^{4x} \cdot 8$$

$$e^{4x} \cdot y' + e^{4x} \cdot 4y = e^{4x} \cdot 8$$

$$\frac{d}{dx}\left(e^{4x} \cdot y\right) = e^{4x} \cdot 8$$

$\int e^u\,du = e^u$

$u = 4x$

$du = 4\,dx$

↓ Integrate

$$e^{4x} \cdot y = \int e^{4x} \cdot 8\,dx = 2\int e^{4x} \underbrace{4\,dx}_{du} = 2e^{4x} + C$$

$$y = \frac{1}{e^{4x}}\left[2e^{4x} + C\right] = 2 + \frac{C}{e^{4x}} = 2 + C \cdot e^{-4x}$$

$$\boxed{y = 2 + Ce^{-4x}}$$

gen'l solution

check

given: $y' + 4y = \underline{\underline{8}}$, substitute / differentiate

take derivative of answer

$$(-4Ce^{-4x}) + 4(2 + Ce^{-4x}) \stackrel{?}{=} -4Ce^{-4x} + 8 + 4Ce^{-4x} = 8$$

simplify

:)

Solve the Initial Value Problem: when $x = 1$, $y = 2$

$$y' + \frac{4}{x}y = 8 \quad // \quad y(1) = 2$$

① $e^{\int 4/x\,dx} = e^{4\int 1/x\,dx} = \left(e^{\int 1/x\,dx}\right)^4 = \left(e^{\ln|x| + c}\right)^4 = \left(|x| \cdot e^c\right)^4 = x^4 \cdot C_1$

drop C_1 (need one sol'n)

$e^{AB} = (e^B)^A$

$$\boxed{u(x) = x^4}$$

② $x^4\left(y' + \frac{4}{x}y\right) = 8x^4$

$\quad\quad \| $

$x^4 y' + 4x^3 y$

$\quad\quad \| $

$\frac{d}{dx}(x^4 \cdot y) = 8x^4$

③ Integrate

$x^4 y = \int 8x^4\,dx = \frac{8x^5}{5} + C$

$$\boxed{y = \frac{8}{5}x + \frac{C}{x^4}}$$

gen'l sol'n

④ Initial Value

$2 = \frac{8}{5} + \frac{C}{1}$

$\frac{10-8}{5} = C$

$\frac{2}{5} = C$

$$\boxed{y = \frac{8}{5}x + \frac{2}{5}\left(\frac{1}{x^4}\right)}$$