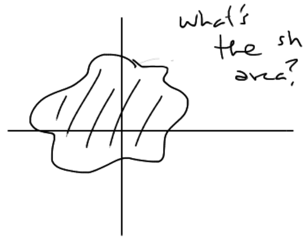
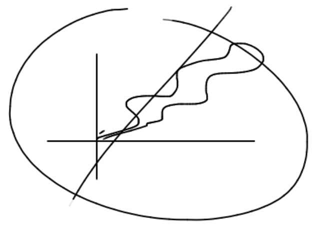
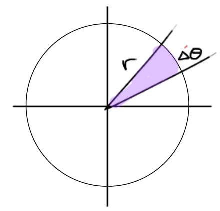
# 11-4 Area & Arc length in Polar Coordinates

## Area:



what's the shaded area?






Recall, that a circular sector of angle $\Delta\theta$ with radius has area $\frac{1}{2}r^2 \cdot \Delta\theta$

why? Area of circle $A = \pi r^2$

for angle $\theta$, the sector defined by $\theta$ has area proportional to $\theta$

if $\theta = 2\pi \Rightarrow$ whole circle $A = \pi r^2$

$\theta = \pi \Rightarrow$ ½ circle $A = \pi r^2 \cdot \frac{\pi}{2\pi} = \frac{1}{2}\pi r^2$

etc.,

$$A_\theta = \pi r^2 \cdot \frac{\theta}{2\pi}$$

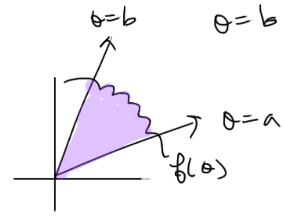
algebra

$$A_\theta = \frac{1}{2} \cdot r^2 \cdot \theta$$

Area of region $\approx \sum_{i=1}^{N} \frac{1}{2} r_i^2 \cdot \Delta\theta$ ($= f(\theta)$) $= \sum_{i=1}^{N} \frac{1}{2} f^2(\theta)\,\Delta\theta$

Exact Area $= \lim_{N\to\infty} \sum_{i=1}^{N} \frac{1}{2} f^2(\theta)\,\Delta\theta = \boxed{\frac{1}{2}\int_a^b f^2(\theta)\,d\theta}$

Area bounded by $r = f(\theta)$ & the rays $\theta = a$, $\theta = b$



**Ex:** circle w/ radius = 2 (Compute Area)

Rect Formula: $x^2 + y^2 = 4$

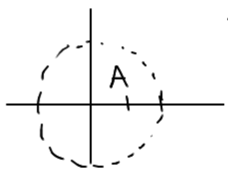
Polar Formula: $(r\cos\theta)^2 + (r\sin\theta)^2 = 4$

use $x = r\cos\theta$

$y = r\sin\theta$

$r^2\cos^2\theta + r^2\sin^2\theta = 4$

$r = 2$




By symmetry Total Area = 4A

A is bound by rays $\theta = 0$, $\theta = \frac{\pi}{2}$ & $f(\theta) = r = 2$

$$A = \frac{1}{2}\int_0^{\frac{\pi}{2}} (r)^2 d\theta = \frac{1}{2}\int_0^{\pi/2} 4\, d\theta = 2\int_0^{\pi/2} d\theta = 2\theta\Big|_0^{\pi/2}$$

$$= \frac{2\pi}{2} = \pi.$$

Total = $4A = 4\pi$

jives w/ known formula ($\pi(r^2) = \pi \cdot 2^2 = 4\pi$)

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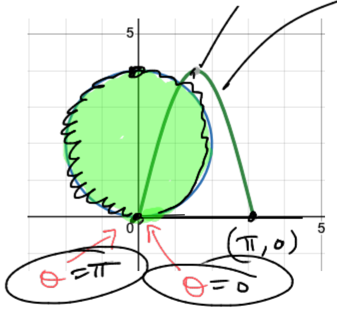
$r = 4\sin\theta$

when $\theta = \frac{\pi}{2}$, $r = 5$

$(\frac{\pi}{2}, 5)$

graph $r$ vs. $\theta$ on rectangular coords




Compute shaded area ____

$$A = \frac{1}{2}\int_0^{\pi} (f(\theta))^2 d\theta = \int_0^{\pi} (4\sin\theta)^2 d\theta = \cancel{16}^{\,8}\int_0^{\pi} \sin^2\theta\, d\theta$$

$$\boxed{\begin{aligned}\sin^2\theta &= \frac{1}{2}(1 - \cos(2\theta))\\ \cos^2\theta &= \frac{1}{2}(1 + \cos(2\theta))\end{aligned}}$$

integral of sin-squared is VERY common with these problems (cos-squared too)

$$= \cancel{16}^{\,8}\int_0^{\pi} \frac{1}{2}(1 - \cos(2\theta))d\theta = 8\int_0^{\pi} d\theta - 8\int_0^{\pi} \cos(2\theta)\, d\theta$$

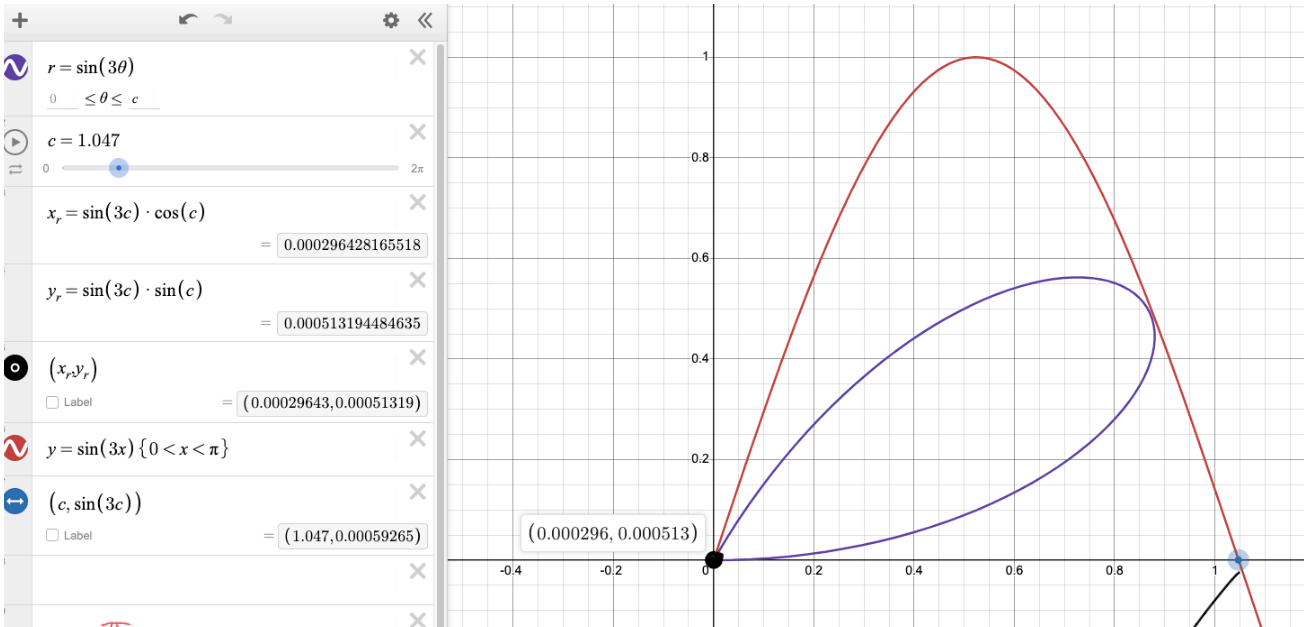
$u = 2\theta$

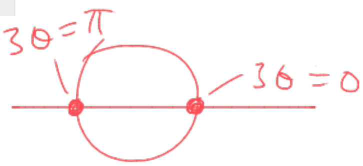
$\frac{1}{2} du = d\theta$

$$= \cancel{8}^{\,4}\theta\Big|_0^{\pi} - \cancel{8}^{\,4}\int_0^{2\pi} \cos(u)\, du \qquad = -8\int_0^{2\pi} \cos(u)\, du$$

$$= \cancel{8}^{\,4}\pi - \cancel{8}^{\,4}[\sin(2\pi - \sin(0)] = \cancel{8}^{\,4}\pi = 4\pi$$

$r = \sin(3\theta)$




$3\theta = \pi$

$\theta = \frac{\pi}{3}$

$r = \sin 3\theta = 0$

$\sin 3\theta = 0$

$3\theta = \sin^{-1}(0) = 0$

$\theta = 0$

$$\text{Petal Area} = \int_0^{\pi/3} \sin^2(3\theta)\, d\theta = \frac{1}{2}\int_0^{\pi/3} (1 - \cos(6\theta))\, d\theta$$