$$\sin 2x = 2\sin x\cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos^2\theta = \left(\frac{1}{2}\right)(1+\cos 2\theta)$$
$$\sin^2\theta = \left(\frac{1}{2}\right)(1-\cos 2\theta)$$

$$\sin A\cos B = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$$
$$\sin A\sin B = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$$
$$\cos A\cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$$

Taylor / Maclaurin series generation:

$$\sum_{n=0}^{+\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$\begin{aligned}
0! &= 1\\
1! &= 1\\
2! &= 2\\
3! &= 6\\
4! &= 24\\
5! &= 120\\
6! &= 720\\
7! &= 5040\\
8! &= 40320\\
9! &= 362880
\end{aligned}$$

scratch paper

163 Final Exam

1. Integrals . . . .
(a)

$$\int 4x^3 \cos(x^4 - 5)\, dx$$

(b)

$$\int x \sin x \, dx$$

(c)

$$\int \frac{4x - 5}{x^2 - 7x - 8}\, dx$$

(d)

$$\int \tan^3\theta \sec\theta \, d\theta$$

(e)

$$\int \frac{x^3}{\sqrt{1+x^2}} \, dx$$

Pick 3 (or more)

(f)
$$\int \sin^{-1} x \, dx$$

(g)
$$\int_0^{+\infty} e^{-4x} \, dx$$

(h)
$$\int e^{4x} \sin x \, dx$$

(i)
$$\int \sqrt{4 - x^2} \, dx$$

(j)
$$\int \sec^3 \theta \, d\theta$$

(k)
$$\int \frac{2x^3 + x^2 + 4x + 1}{(x^2 + 1)^2} \, dx$$

2. Series (& Sequences) . . . .

(a) determine convergence/divergence

$$\sum_{k=1}^{\infty} \frac{\sqrt{k}}{3k+4}$$

(b) determine convergence/divergence

$$\sum_{k=1}^{\infty} \frac{\sqrt[3]{k}}{4k^2+9}$$

(c) determine convergence/divergence

$$\sum_{k=0}^{\infty} \frac{k!}{2^{k^2}}$$

(d) determine convergence/divergence

$$\sum_{k=1}^{+\infty} \sqrt{\frac{25k}{100+k}}$$

(e) is it absolutely convergent, conditionally convergent, or divergent? (Justify your answer.)

$$\sum_{k=1}^{\infty} (-1)^k \frac{\sqrt[3]{k}}{4k^2+9}$$

(f) find the value of the series . . . .

$$\sum_{k=2}^{\infty} \frac{9}{2^k}$$

(g) find the value of the series . . . .

$$\sum_{k=2}^{+\infty}[64^{1/k} - 64^{1/(k+2)}]$$

(h) examples . . . .

Give three examples of a *sequence* that converges to $\ln 5$.

Give an example of a divergent sequence.

3. Taylor & Maclaurin stuff . . . .

Find the fourth degree Taylor polynomial for the function $f(x) = \ln(x-1)$ at $x = 2$.

4. More Taylor & Maclaurin stuff . . . .

(a) Find the interval of convergece for the power series below:

$$\gamma(x) = x - \frac{1}{4}x^4 + \frac{1}{7}x^7 - \cdots = \sum_{n=0}^{+\infty} \frac{(-1)^n}{3n+1} x^{3n+1}$$

(b) Find the limit:

$$\lim_{x \to 0} \frac{\gamma(x) - x}{x^4}$$

(c) Use a seventh degree polynomial to estimate

$$\int_0^1 \gamma(x)\, dx$$

5. Show that if $\sum_{n=0}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

6. Find the Maclaurin series representation of the function $f(x) = e^x$. (Show your work - don't just throw down a power series . . . .)