Name:
Math 163 Final Exam Guide
Date: April 21, 2025

Integration

1. $$\int 4x^3 \cos(x^4 - 5)dx$$

2. $$\int x \sin x dx$$

3. $$\int \frac{4x - 5}{x^2 - 7x - 8}dx$$

4. $$\int \frac{x^3}{\sqrt{1 + x^2}}dx$$

5.

$$\int \tan^3(\theta)\sec(\theta)d\theta$$

6.

$$\int \sin^{-1}(x)dx$$

7.

$$\int_0^{+\infty} e^{-4x}dx$$

8.

$$\int e^{4x}\sin x dx$$

9.

$$\int \sqrt{4-x^2}dx$$

10.

$$\int \sec^3(x)dx$$

Sequences & Series

11. Determine whether the series converges or diverges:

(11.1)

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

(11.2)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

(11.3)

$$\sum_{n=1}^{\infty} \frac{1}{n!}$$

(11.4)

$$\sum_{n=1}^{\infty} \frac{2^n}{3^n}$$

(11.5)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

(11.6)

$$\sum_{k=1}^{\infty} \frac{\sqrt{k}}{3k+4}$$

(11.7)

$$\sum_{k=1}^{\infty} \frac{\sqrt[3]{k}}{4k^2+9}$$

(11.8)

$$\sum_{k=1}^{\infty} \frac{k!}{2^k}$$

(11.9)

$$\sum_{k=1}^{\infty} \sqrt{\frac{25k}{100+k}}$$

(11.10)

$$\sum_{k=1}^{\infty} (-1)^k \frac{\sqrt[3]{k}}{4k^2+9}$$

(11.11)

$$\sum_{k=1}^{\infty} \frac{9}{2^k}$$

(11.12)

$$\sum_{k=1}^{\infty} [64^{1/k} - 64^{1/(k+2)}]$$

(11.13) Give three examples of a *sequence* that converges to $\ln(5)$.

(11.14) Give three examples of a divergent sequence.

Taylor & Maclaurin

1. Taylor:

(1.1) Find fourth degree Taylor polynomial for the function

$$f(x) = \ln(x-1) \text{ at } x = 2$$

.

(1.2) Find the interval of convergece for the power series below:

$$\gamma(x) = x - \frac{1}{4}x^4 + \frac{1}{7}x^7 - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{3n+1} x^{3n+1}$$

(1.3) Using the $\gamma(x)$ from the previous problem find the limit

$$\lim_{x \to 0} \frac{\gamma(x) - x}{x^4}$$

(1.4) Use a seventh degree polynomial to estimate

$$\int_0^1 \gamma(x) dx$$

(1.5) Show that if $\sum_{n=0}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

(1.6) Find the Maclaurin series representation of the function $f(x) = e^x$. (Show your work, i.e., derive the series from scratch)

Parametric, Polar & Differential Equations

5. Find the area of the region bounded by the curve $r = 2 + 2\sin\theta$ and the line $\theta = \frac{\pi}{3}$.

6. Compute the arc length of a circle centered at the origin of radius 4 using polar coordintes.

7. Solve the following initial value problem:

$$x\frac{dy}{dx} = y - 2x, y(2) = \pi$$

8. Solve the following initial value problem:

$$\frac{dy}{dx} + 4y = e^{-4t}, y(0) = 4$$

9. Solve the following initial value problem:

$$yy' = xe^{-y^2}, y(0) = -5$$

10. Solve the following initial value problem:

$$(1 - 9t)\frac{dy}{dt} - y = 0, y(2) = -6$$