10 am section below

1pm section at end

MA161 – Week 14 – Monday

Take-Home:

- Wed classtime or Thurs classtime
- Questions

  #2: Hint: It's a mistake to get 0 for your answer (length)

you'll see something like $\displaystyle\int_0^{2\pi} \sqrt{\sin^2(t)} = \int_0^{2\pi} |\sin(t)|\,dt$

$$= \int_0^{\pi} \sin(t)\,dt + \int_{\pi}^{2\pi} -\sin(t)\,dt$$

ALTERNATIVELY:

Total Length = 4 times length of one arc

 $= 4 \times$  $\displaystyle = 4\int_0^{\pi/2} \sin(t)\,dt$

Last Week

Review all week

- Monday - Exam 1 / General overview of Course

Final Exam Covers:

Exam 1	Exam 2	Exam 3
Integration	Sequences / Series	Power Series / Taylor Series
def'n:		

Integration

Def'n: The Indefinite Integral: $\int f(x)\,dx = F(x)$ w/ $F'(x) = f(x)$

Def'n: the definite Integral: $\int_a^b f(x)\,dx$ = Area under graph of curve $y=f(x)$

Fundamental Theorem of Calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$

Techniques:

- u-sub / memorizing forms (1st try, identifying derivative relationships)
  degree-1 difference
- Trig sub / Int. by parts / partial fractions

Common Forms

$u = u(x)$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C$$

$$\int \sin(u)\,du = -\cos(u) + C$$

$$\int \cos(u)\,du = \sin(u) + C$$

$$\int \tan(u)\,du = -\ln|\cos(u)| + C = \ln|\sec(u)| + C$$

$\tan(u) = \frac{\sin(u)}{\cos(u)}$

$$\int \sec(u)\tan(u)\,du = \sec(u) + C$$

$$\int \sec^2(u)\,du = \tan(u) + C$$

$$\int \sec(u)\,du = \ln|\sec(u) + \tan(u)| + C$$

(multiply top/bottom by $\sec(u) + \tan(u)$)

$$\int e^u\,du = e^u + C$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1}(u) + C$$

$$\int \frac{-1}{\sqrt{1-u^2}}\,du = \cos^{-1}(u)$$

$$\int \frac{1}{1+u^2}\,du = \tan^{-1}(u)$$

$$\int \frac{1}{u\sqrt{u^2-1}}\,du = \sec^{-1}(u)$$

These can be found by:

start: $y = \tan^{-1}(u)$

- goal: find $y' = \frac{dy}{dx}$
- $\tan(y) = u$
- $\frac{d}{dx}(\tan(y)) = \frac{d}{dx}(u)$

chain rule

$\sec^2(y) \cdot y' = \frac{du}{dx}$

- $y' = \frac{1}{\sec^2(y)} \cdot \frac{du}{dx}$

$= \frac{1}{\tan^2\theta + 1} \cdot \frac{du}{dx}$

$= \frac{1}{u^2+1} \cdot \frac{du}{dx}$

$\sin^2\theta + \cos^2\theta = 1$

$\div \cos^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

Ex

$$\int e^{3x+5}\,dx = \int e^{u}\cdot\frac{1}{3}\,du = \frac{1}{3}\int e^{u}\,du = \frac{1}{3}e^{u}+C = \frac{1}{3}e^{3x+5}+C$$

$u = 3x+5$

$$\frac{d}{dx}(u) = \frac{du}{dx} = 3 \sim du = 3\,dx \rightarrow \frac{1}{3}\,du = dx$$

Ex

$$\frac{1}{2}\int 2\cancel{x}\cdot\sec(e^{x^2})\tan(e^{x^2})\cdot\cancel{e^{x^2}}\,\cancel{dx} = \frac{1}{2}\int \sec(u)\tan(u)\,du$$

$$= \frac{1}{2}\sec(e^{x^2})+C$$

$u = e^{x^2}$

$du = \cancel{e^{x^2}}\cdot 2\cancel{x}\,\cancel{dx}$

Ex

$$\int \frac{1}{x\sqrt{x^6-1}}\,dx$$

① similar to $\int \frac{1}{u\sqrt{u^2-1}}\,du$

② think: $\boxed{u = x^3} \sim u^2 = x^6$

$du = 3x^2\,dx$

$\boxed{\frac{1}{3x^2}\,du = dx}$

$$= \int \frac{1}{\cancel{x}\sqrt{u^2-1}}\cdot\frac{1}{3\cancel{x^2}}\,du$$

$$\frac{1}{3}\int \frac{1}{u\sqrt{u^2-1}}\,du$$

$$= \frac{1}{3}\sec^{-1}(u)+C = \frac{1}{3}\sec^{-1}(x^3)+C$$

Trig sub / int by parts

(From study guide #1)

$$\int \sin^5\theta \, d\theta = \int \sin\theta \cdot \sin^2\theta \cdot \sin^2\theta \, d\theta$$

odd power

$$= \int \sin\theta \,(1-\cos^2\theta)(1-\cos^2\theta)\, d\theta$$

FOIL

$$= \int \sin\theta\,(1 - 2\cos^2\theta + \cos^4\theta)\, d\theta$$

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$-\frac{1}{\sin\theta}\, du = d\theta$

$$= -\int (1 - 2u^2 + u^4)\, du$$

$$= -\left(u - \frac{2}{3}u^3 + \frac{u^5}{5}\right) + C = -\left(\cos\theta - \frac{2}{3}\cos^3\theta + \frac{\cos^5\theta}{5}\right) + C$$

Trig Sub patterns:

$$\int \sqrt{1-x^2}\, dx \qquad \int \frac{1}{\sqrt{1-x^2}}\, dx$$

(u-sub fails)

$x = \sin\theta$

domain of given

range of sub

$$\int \sqrt{x^2-1}\, dx \qquad \int \frac{1}{\sqrt{x^2-1}}\, dx$$

$x = \sec\theta$

$$\int \sqrt{1+x^2}\, dx \qquad \int \frac{1}{\sqrt{1+x^2}}\, dx$$

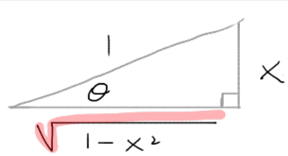
$x = \tan\theta$

$$\int \sqrt{1-x^2}\, dx$$

$x = \sin\theta$

$dx = \cos\theta \, d\theta$

$\theta = \sin^{-1} x$



$\sin\theta = \frac{x}{1} = x$

$\cos\theta = \sqrt{1-x^2}$

$$= \int \cos^2\theta \, d\theta$$

$$= \int \frac{1+\cos 2\theta}{2}\, d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C$$

$2\sin\theta \cdot \cos\theta$

$$= \frac{1}{2}\sin^{-1}x + \frac{1}{4}\cdot 2 \cdot x \cdot \sqrt{1-x^2} + C$$

$$= \boxed{\frac{1}{2}\sin x + \frac{1}{2}x\sqrt{x^2-1} + C}$$

1pm section below

MA1G3 - Week 14 - Monday

Take-Home Exam:

1. Wed classtime or Thurs. Classtime
2. 93 - thing: Calculated after Take-Home,
3. Questions: differential equations

Final Exam:

Differential Equations:

separable

$y' = f(x) \cdot g(y)$

$y(0) = 1$

Initial Value Problem

find precise c that makes it work

$\boxed{\begin{array}{c} 1^{st} \text{ order linear} \\ y' + p(x)\,y = q(x) \\ y(1) = 5 \end{array}}$

Ex. Solve the I.V.P.

$y' = (x+1)\cdot(y-3)$

$y(2) = 33$

1. separate!

$$\frac{y'}{y-3} = x+1$$

$y' = \frac{dy}{dx}$ & integrate wrt x

$$\int \frac{\cancel{dy}}{y-3}\cdot\frac{dy}{\cancel{dx}}\cdot \cancel{dx} = \int x+1 \, dx$$

$$\int \frac{dy}{y-3} = \int x+1\, dx \ \ldots$$

Integrating

$$\ldots \ \ln|y-3| = \frac{x^2}{2} + x + C$$

$e^\wedge \qquad e^\wedge$

$$|y-3| = e^{\frac{x^2}{2}+x+C} = e^{\frac{x^2}{2}+x} \cdot e^{C}$$

since $e^C > 0$ let $C_1 = \pm e^C$ to account for $|y-3|$

$$\boxed{y = C_1 \cdot e^{\frac{x^2}{2}+x} + 3}$$

gen'l sol'n

I.V.P $x=2 \Rightarrow y=3$

$33 = C_1 \cdot e^4 + 3$

$30 = C_1 \cdot e^4$

$30e^{-4} = C_1$

$$y = 30e^{-4}\cdot e^{\frac{x^2}{2}+x} + 3$$

$$y = 30\,e^{x^2/2 + x - 4} + 3$$

Ex. $y' - x^2 y = 3x$

$y(1) = 2$

F.O.L $\Rightarrow$ Integrating Factor! $e^{\int p(x)\,dx}$, multiply both sides

① $e^{\int -x^2 dx} = e^{-x^3/3}$ (ignore c) or $c = 0$

② $e^{-x^3/3}(y' - x^2 y) = e^{-x^3/3} \cdot 3x^2$

$e^{-x^3/3} \cdot y' - e^{-x^3/3} \cdot x^2 y =$

product rule

③ $\frac{d}{dx}\left(y \cdot e^{\frac{-x^3}{3}}\right) = e^{-\frac{x^3}{3}} \cdot 3x^2$

Not a u-sub...

I.B.P ______

$\star$

④ goal: find y, integrate $\int LHS\,dx = \boxed{y \cdot e^{\frac{-x^3}{3}} = -3e^{\frac{-x^3}{3}} + C}$

$\star$ $\int RHS = 3\int e^{-\frac{x^3}{3}} \cdot x^2\,dx$

$\boxed{uv - \int v\,du = \int u\,dv}$

$u = -\frac{x^3}{3}$

$du = -x^2\,dx$

$-\frac{1}{x^2}\,du = dx$

$= -3\int e^u\,du$

$= -3e^{\frac{-x^3}{3}} + c$

⑤ $y = -3 + C \cdot e^{\frac{x^3}{3}}$

I.V.P $y = 2$ if $x = 1$

$2 = -3 + C \cdot e^{1/3}$

$\frac{5}{e^{1/3}} = 5e^{-1/3} = c$

⑥ $y = 5e^{-1/3}e^{x^3/3} - 3$

$$\boxed{y = 5e^{\frac{x^3 - 1}{3}} - 3}$$

FINAL EXAM: TRIG SUB PATTERNS

$$\int \sqrt{1+x^2}\,dx \qquad \int \frac{1}{\sqrt{1+x^2}}\,dx \qquad \int \sqrt{4+x^2}\,dx$$

domain

$$x = \begin{cases} \sin\theta & (-1,1) \\ \sec\theta & (-\infty,-1)\cup(1,\infty) \\ \tan\theta & (-\infty,\infty) \end{cases} \longleftarrow \text{range}$$

$x = a\sin\theta$

$$\int \sqrt{a^2 - x^2}\,dx \quad \text{or} \quad \int \sqrt{1-x^2}\,dx \qquad x = \sin\theta$$

$x = a\sec\theta$

$$\int \sqrt{x^2 - a^2}\,dx \quad \text{or} \quad \int \sqrt{x^2-1}\,dx \qquad x = \sec\theta$$