Mon wk 14

Integration

Your Integral

$\int \frac{\sqrt{x+1}}{x}\,dx$

algebra
u-sub

$\int \cos(2x)\,dx$

Integration by Parts

$uv - \int v\,du$

LIATE

Types:

products : $\int e^x \cos x\,dx$

inverse trig : $\int \sin^{-1} x\,dx$

logs : $\int \ln(x)\,dx$

trig integrals

Types $\int \sin^m x \cos^n x\,dx$

$\int \tan^m(x) \sec^n(x)\,dx$

Hint: exploit

(✱) $\sin^2 x + \cos^2 x = 1$

Ex $\int \sin^{odd}(x) \cos^{even}(x)\,dx$

$= \int \sin^{even} \cdot \cos^{even} \quad \sin(x)\,dx$

(✱)

$= \int (1 - \cos^{even}) \cos^{even} \cdot \sin x\,dx$

$u = \cos x$

u-sub

trig substitution

$a^2 - x^2$ $\quad x^2 + a^2$ $\quad x^2 - a^2$

$\sqrt{a^2 - x^2}$ $\quad \sqrt{x^2 + a^2}$ $\quad \sqrt{x^2 - a^2}$

Challenge: what to substitute

domain $\longleftrightarrow$ range
$\sqrt{\ }$ trig fcn

eg $\text{domain}(\sqrt{a^2 - x^2}) = (-a, a)$

trig fcn: $x = a\sin\theta$

eg. $\sqrt{x^2 + a^2}$ ⇝ domain $= \mathbb{R}$

$x = a\tan\theta$

eg $\sqrt{x^2 - a^2}$ ⇝ domain $> a$ or $< -a$

$x = a\sec\theta$

Partial Fractions

– rational w/ bottom factors

Integration by Parts —————— [use everything]

LIATE
LIPET

$\int e^x \cos x \, dx$

$u = \cos x \qquad dv = e^x dx$

$du = -\sin x \, dx \qquad v = e^x$

$= e^x \cos x + \int e^x \sin x \, dx \; = \; \underbrace{e^x \cos x - e^x \cos x}_{=0} - \int (-\cos x) e^x \, dx$

$\begin{cases} u = e^x & dv = \sin x \, dx \\ du = e^x dx & v = -\cos x \end{cases}$ $\qquad = \int e^x \cos x \, dx$ $\qquad$ dead end

instead of this

$u = \sin x \qquad dv = e^x dx$

$du = \cos x \qquad v = e^x$

$= e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$

$\int e^x \cos x \, dx \; = \; e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$

add

$$\int e^x \cos x \, dx = \frac{1}{2}(e^x \cos x + e^x \sin x) + C$$

$\int \sin^{-1} x \, dx = x \sin^{-1} x - \underbrace{\int \frac{x}{\sqrt{1-x^2}} dx}_{u\text{-sub}} = x \sin^{-1} x + \underbrace{\frac{1}{2} \int}\frac{du}{\sqrt{u}}$

$u = \sin^{-1} x \qquad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}} \qquad v = x$

u-sub

$u = 1 - x^2$

$du = -2x \, dx$

$-\frac{1}{2} du = x \, dx$

$\frac{1}{2} \int u^{-1/2} = \frac{1}{2} \frac{u^{1/2}}{\frac{1}{2}}$

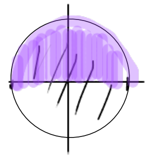
$= \sqrt{u}$

$= x \sin^{-1} x + \sqrt{1-x^2} + C$

same idea for $\int \ln x \, dx$

You've learned a lot this semester...

In Calc I, $A = \pi r^2$ was ok, but deriving it via integrals produced:



$x^2 + y^2 = r^2$

$y = \sqrt{r^2 - x^2}$

set $r = 4$

$$2\int_{-4}^{4} \sqrt{16 - x^2}\, dx$$

set $x = 4\sin\theta$

$dx = 4\cos\theta\, d\theta$

$x = -4 = 4\sin\theta$

$-1 = \sin\theta$

$-\frac{\pi}{2} = \theta$

$$\sqrt{16(\underbrace{1 - \sin^2\theta}_{\cos^2\theta})} = \sqrt{16}\cdot\sqrt{\cos^2\theta} = 4|\cos\theta|$$

$$= 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{16 - 16\sin^2\theta}\cdot 4\cos\theta\, d\theta$$

$$2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4|\cos\theta|\cdot 4\cos\theta\, d\theta = 32\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta = 32\int \tfrac{1}{2}(1 + \cos(2\theta))$$

$$= 16\int (1 + \cos(2\theta))\, d\theta = 16\int_{-\frac{\pi}{2}}^{\pi/2} 1\, d\theta + \underbrace{16\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(2\theta)\, d\theta}_{=0}$$

$$= 16\theta\Big|_{-\pi/2}^{\pi/2} = 16\left(\pi/2 - (-\pi/2)\right) = 16\pi \qquad \pi r^2$$

In Polar Coords:

$r = 4$

$$A = \frac{1}{2}\int_0^{2\pi} r^2\, d\theta = \frac{1}{2}\int_0^{2\pi} 16\, d\theta = 8\int_0^{2\pi} d\theta = 8\theta\Big|_0^{2\pi} = 16\pi$$