Mon Wk 14

Integration:

TOOLS: u-sub, integration by parts (multiple times) (product rule for ints.), partial fractions, trig-integrals, trig-sub, Improper int $\int_a^{\infty} f(x)\,dx$

Your Integral

algebra / u-sub: $\int \frac{x+1}{\sqrt{x}}\,dx$, $\int \cos(2x)\,dx$

I.B.P. LIATE

- products: $\int e^x \cdot \cos x\,dx$
- inverse trig: $\int \sin^{-1}x\,dx$
- logs: $\int \ln x\,dx$

$uv - \int v\,du$

TRIG INT

sin, cos, sec, etc,

$\sin^2 + \cos^2 = 1$

$\cos^2 x = \frac{1}{2}(1+\cos\frac{x}{2})$

$\sin^2 x = \frac{1}{2}(1-\cos\frac{x}{2})$

TRIG SUB

$a^2 - x^2$, $x^2 + a^2$, $x^2 - a^2$

- $\sqrt{a^2 - x^2}$ ⇝ use domain ∈ $[-a, a]$ ⟷ range $x = a\sin(\theta)$ to match which trig function
- $\sqrt{x^2 + a^2}$ ← domain $= \mathbb{R}$ ⇒ want range $= \mathbb{R}$ $x = \tan(\theta)$
- $\sqrt{x^2 - a^2}$ ← domain: $(-\infty, a] \cup [a, \infty)$ ⇒ $x = \sec(\theta)$

Ex Area of unit circle 2π

$$2\int_0 \sqrt{1 - x^2}\,dx$$

Partial Fractions

rational where bottom factors

Ex (Everything gets used)

$\int e^x \cos x\,dx =$

$u = \cos x$ $dv = e^x dx$

$du = -\sin x\,dx$ $v = e^x$

$= \cos x\, e^x + \int e^x \sin x\,dx$

$u = \sin x$ $dv = e^x$

$du = \cos x\,dx$ $v = e^x$

$= \cos x\, e^x + \sin x\, e^x - \int e^x \cos x\,dx$

$$\int e^x \cos x\,dx = \frac{1}{2}\left[e^x[\cos x + \sin x]\right] + C$$

Ex $\int \sin^{-1}x\,dx$

$u = \sin^{-1}x$ $dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}dx$ $v = x$

$= x \cdot \sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}dx$

$u = 1 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= x\sin^{-1}x + \frac{1}{2}\int u^{-1/2}du$

$= x\sin^{-1}x + (1-x^2)^{\frac{1}{2}} + C$

Ex.

$\int \ln x\,dx = x\ln x - \int \frac{x}{x}dx$

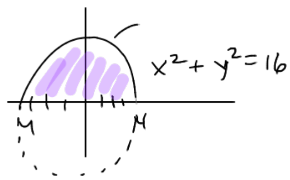
$u = \ln x$ $dv = dx$

$du = \frac{1}{x}dx$ $v = x$

$= x\ln x - \int 1\,dx = x\ln x - x + C$

Note! You've learned ways to make some calculations much easier than before.

Find the area of a circle w/ radius 4 via integration.



$$A = 2\int_{-4}^{4}\underbrace{\sqrt{16-x^2}}\,dx = 2\int_{-\pi/2}^{\pi/2}\sqrt{16-16\sin^2\theta}\cdot 4\cos\theta\,d\theta$$

$4^2 - x^2 \Rightarrow$

$$\boxed{\begin{array}{l} x = 4\sin\theta \\ dx = 4\cos\theta\,d\theta \\ x = 4 = 4\sin\theta \\ \sin\theta = 1 \\ \theta = \pi/2 \end{array}}$$

$\sqrt{16-16\sin^2\theta}$

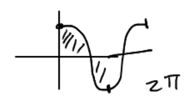
$\sqrt{16(1-\sin^2\theta)}$

$4\sqrt{\cos^2\theta}$

$4|\cos\theta|$

$\cos > 0$ on $(-\frac{\pi}{2}, \frac{\pi}{2})$

$$= 8\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4|\cos\theta|\cdot\cos\theta\,d\theta = 32\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2\theta\,d\theta = 32\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\tfrac{1}{2}(1+\cos 2\theta)\,d\theta$$



$$= 16\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1\,d\theta + 16\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos(2\theta)\,d\theta = 16\theta\Big|_{-\frac{\pi}{2}}^{\pi/2} + \overbrace{32\underbrace{\int_{-\pi}^{\pi}\cos(u)\,du}}^{=0}$$

$u = 2\theta$

$du\ 2\,d\theta$

$\frac{1}{2}du = d\theta$

$\theta = \frac{\pi}{2} \Rightarrow u = \pi$

$\theta = -\frac{\pi}{2} \Rightarrow u = -\pi$

$\sin(u)\Big|_{-\pi}^{\pi} = \sin(\pi) - \sin(-\pi)$

$= 0$

$$= 16\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = 16\pi$$

$\pi 4^2$

ALT Circle of $r = 4$ in Polar $r = 4$

Polar Area Formula $= \frac{1}{2}\int_0^{2\pi} r^2\,d\theta = \frac{1}{2}\int_0^{2\pi} 16\,d\theta = 8\int_0^{2\pi} d\theta = 8\theta\Big|_0^{2\pi} = 16\pi$