Wk 14 Wed

Tomorrow: Exam 4 due + Presentation

Study Guide + Sols: Posted Today / Soon

How to submit work on Desmos:

① Take Screenshot + Print

② Recopy: From Desmos to paper.

Ex $\int_a^b f(x)\,dx = 5.3$ (Desmos)

Today: Sequences & Series

Basics: Sequence: ordered list of numbers (converge ≡ limit of terms exists)

Series: sum of terms (converge ≡ sequence of partial sums converge)

Ex ① work out sequence of partial sums (get formula)

(11.12) ② take limit

$$\sum_{k=1}^{\infty}[64^{1/k} - 64^{1/(k+2)}] -$$

Mostly though, we only ask whether series converges or diverges.

Summary of Convergence / Divergence Tests for Series

Test	Series	Converges	Diverges	Comments	Example
divergence test	$\sum_{n=1}^{\infty} a_n$	N/a	$\lim_{n\to\infty} a_n \neq 0$	apply First	$\sum_{k=1}^{\infty}\sqrt{\frac{25k}{100+k}}$ diverges b/c $\lim_{n\to\infty} a_n = 5 \neq 0$
geometric series	$\sum_{n=0}^{\infty} Cr^n$	$\|r\| < 1$	$\|r\| \geq 1$	common ratio	$\sum_{n=0}^{\infty} 5\left(\frac{1}{3}\right)^n$ converges vs $\sum_{n=0}^{\infty} 5\left(\frac{4}{3}\right)^n$ diverges
P-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	HARMONIC : $p = 1$	$\sum_{n=1}^{\infty} \frac{1}{n}$ = diverges
Integral Test	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n)$	$\int_1^{\infty} f(x)dx$ conv.	$\int_1^{\infty} f(x)dx$ div	$f(x)$ = continuous positive decreasing	Harmonic series div b/c Int. Test $\int_1^{\infty}\frac{1}{x}dx = \ln\|x\|\Big\|_1^{\infty} = \lim_{N\to\infty} \ln\|x\|\Big\|_1^N$ $= \lim_{N\to\infty} \ln\|N\| - \underset{0}{\ln(1)}$ $= \infty$
D.C.T. direct comparison	$\sum_{n=1}^{\infty} a_n$	$0 \leq a_n \leq b_n$ $\sum b_n$ converges	$0 \leq b_n \leq a_n$ $\sum b_n$ div	suspect conv: Find bigger series that conv. Suspect div: Find smaller series that div.	$\sum_{n=0}^{\infty}\frac{1}{n+1}$ = div. b/c : $\underbrace{\frac{1}{n} < \frac{1}{n+1}}_{\text{False}}$ & $\sum \frac{1}{n}$ div False ⇒ D.C.T. does not apply
L.C.T. limit compare t	$\sum_{n=1}^{\infty} a_n$	$\underbrace{\lim\left\|\frac{a_n}{b_n}\right\| > 0}_{\text{behave similarly}}$ if $\sum b_n$ converges	$\lim\left\|\frac{a_n}{b_n}\right\| > 0$ If $\sum b_n$ div.	Inconclusive if $\lim = 0$ or ∞	$\sum \frac{1}{n-1}$ = div. by D.C.T. $\sum \frac{1}{n+1}$ $b_n = \frac{1}{n}$ $\lim\left\|\frac{\frac{1}{n+1}}{\frac{1}{n}}\right\| = \left\|\frac{n}{n+1}\right\| \to 1$
A.S.T. alt.	$\sum(-1)^n a_n$	① alt. ② dec ⇒ Converge	① alt $\lim_{n\to 0} a_n \neq 0$	Ex $\sum(-1)^n \underset{a_n}{(1)}$ = div b/c $\lim a_n \neq 0$	⇒ $\sum \frac{1}{n+1}$ div since $\sum \frac{1}{n}$ div
Ratio Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty}\left\|\frac{a_{n+1}}{a_n}\right\| < 1$	$\lim\left\|\frac{a_{n+1}}{a_n}\right\| > 1$	= 1 inconclusive	Ex Factorials
Root Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty}\sqrt[n]{\|a_n\|} < 1$	$\lim\sqrt[n]{a_n} > 1$	= inconclusive	Ex power $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^n$

apply root test

$\sqrt[n]{\left(\frac{n}{n+1}\right)^n} = \frac{n}{n+1}$

$\lim_{n\to\infty}\frac{n}{n+1} = 1$

inconclusive

Try ratio test

$\frac{\frac{n+1}{n+2}}{\frac{n}{n+1}} = \frac{n+1}{n+2}\cdot\frac{n+1}{n} \to \frac{n^2}{n^2} \to 1$?

⇒ $\boxed{\text{DIV}}$

Div. Test $\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = e^{-1} \neq 0$

$y = \lim\left(\frac{n}{n+1}\right)^n$

$\ln y = \ln \lim\left(\frac{n}{n+1}\right)^n$

$= \lim_{n\to\infty}\frac{\ln\left(\frac{n}{n+1}\right)}{\frac{1}{n}} = \frac{0}{0}$ L'H

$\lim\frac{\frac{1}{(n+1)^2}}{-\frac{1}{n^2}} = \frac{-n^2}{(n+1)^2} \to -1$

$\ln y = 1$ ⇒ $\boxed{y = e^{-1} \neq 0}$