TRIG INT. TECHNIQUES

$\int \sin^n(x)\cos^m(x)\,dx$ look for odd power ....
u = "other function"

odd $\Rightarrow u = \cos(x)$

eg. $\int \sin^3(x)\cos^4(x) = \int \sin^2(x)\cos^4(x)\sin(x)\,dx$

$(1-\cos^2(x))$

$= -\int (1-u^2)u^4\,du$

$\int \sec^m(x)\tan^n(x)$.

Pythag. IDs

$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$

special integrals

① $\int \sec(x)\,dx = \int \sec(x)\cdot\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}\,dx = \ln|\sec x+\tan x| + C$

② $\int \sec^3(x)\,dx \overset{IBP}{=} \int \sec^2 x\cdot\sec x\,dx = \sec x\tan x - \int \sec\tan^2 x\,dx$

③ $\int \sec^5(x)\,dx$

$dv = \sec^2 x\,dx$, $u = \sec x$

$v = \tan x$, $du = \sec x\tan x\,dx$

$= \sec x\tan x - \int \sec x(\sec^2 x - 1)\,dx$

$= \sec\tan x - \int \sec^3 x - \int \sec x$

add integral of sec^3(x) to both sides

$2\int \sec^3(x)\,dx = \sec\tan x - \int \sec x$

divide by 2

$\int \sec^3(x)\,dx = \frac{\sec\tan x - \int \sec x}{2} = \frac{\sec\tan x - \ln|\sec x+\tan x|}{2} + C$