

some standard trig problems

1.

$$\begin{aligned}\int \cos^2 x \, dx &= \int \frac{1}{2} (1 + \cos 2x) \, dx \\ &= \frac{1}{2}x + \frac{1}{4} \sin 2x + C\end{aligned}$$

2. Use $u = \cos x$

$$\begin{aligned}\int \sin^3 x \, dx &= \int \sin^2 x \sin x \, dx \\ &= \int (1 - \cos^2 x) \sin x \, dx = -\cos x + \frac{1}{3} \cos^3 x + C\end{aligned}$$

3. Use $u = \sin x$

$$\begin{aligned}\int \cos^3 x \sin^2 x \, dx &= \int \cos^2 x \sin^2 x \cos x \, dx \\ &= \int (1 - \sin^2 x) \sin^2 x \cos x \, dx = \int (\sin^2 x - \sin^4 x) \cos x \, dx \\ &= \frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C\end{aligned}$$

4. Use $u = \sin x$

$$\int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx = \ln |\sin x| + C$$

5. Use $u = \cot x + \csc x$

$$\begin{aligned}\int \csc x \, dx &= \int \csc x \left(\frac{\csc x + \cot x}{\csc x + \cot x} \right) \, dx \\ &= \int \frac{\csc^2 x + \csc x \cot x}{\cot x + \csc x} \, dx = -\ln |\cot x + \csc x| + C\end{aligned}$$

6.

$$\begin{aligned}\int \sin^2 x \cos^2 x \, dx &= \frac{1}{2}(1 - \cos 2x) \frac{1}{2}(1 + \cos 2x) \, dx \\ &= \int \frac{1}{4} - \frac{1}{4} \cos^2 2x \, dx = \int \frac{1}{4} - \frac{1}{4} \cdot \frac{1}{2}(1 + \cos 4x) \, dx \\ &= \int \frac{1}{8} - \frac{1}{8} \cos 4x \, dx = \frac{1}{8}x - \frac{1}{32} \sin 4x + C\end{aligned}$$

7.

$$\int \sec x \tan x \, dx = \sec x + C$$

8.

$$\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx = \tan x - x + C$$

9. Use $u = \tan x$

$$\int \sec^2 x \tan^2 x \, dx = \frac{1}{3} \tan^3 x + C$$

10. Use $u = \sec x$

$$\begin{aligned}\int \sec^3 x \tan^5 x \, dx &= \int \sec^2 x \tan^2 x \tan^2 x \sec x \tan x \, dx \\ &= \int \sec^2 x (\sec^2 x - 1) (\sec^2 x - 1) \sec x \tan x \, dx \\ &= \int (\sec^6 x - 2 \sec^4 x + \sec^2 x) \sec x \tan x \, dx \\ &= \frac{1}{7} \sec^7 x - \frac{2}{5} \sec^5 x + \frac{1}{3} \sec^3 x + C\end{aligned}$$