Integration by parts–guidelines

Example:

$$\int x \ln x \, dx = ??$$

First "select" your u and dv.

Selection guidelines: the derivative of u should be less complex than u, your dv should have a relatively simple antiderivative, *and* you need to "use up" everything in the integral sign. For Math 134–following "LIPET" to pick your u normally works:

L $\Longrightarrow$ logarithmic functions
I $\Longrightarrow$ inverse trigonometric functions
P $\Longrightarrow$ polynomials
E $\Longrightarrow$ exponential functions
T $\Longrightarrow$ trigonometric functions

Then find du **and** v *based on your selection.*

$$u = \ln x \qquad\qquad dv = x\,dx$$

$$du = \frac{1}{x}\,dx \qquad\qquad v = \int x\,dx = \frac{1}{2}x^2$$

Now apply the integration by parts rule $\int u\,dv = uv - \int v\,du$ to get

$$\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 * \frac{1}{x}\,dx$$

So

$$\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x\,dx = \frac{1}{2}x^2 \ln x - \left(\frac{1}{2}\right)\left(\frac{x^2}{2}\right) + \text{C} = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + \text{C}$$

Check:

$$\frac{d}{dx}\left(\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2\right) = \frac{d}{dx}\left(\frac{1}{2}x^2 \ln x\right) - \frac{d}{dx}\left(\frac{1}{4}x^2\right)$$

$$= x \ln x + \left(\frac{1}{2}x^2\right)\left(\frac{1}{x}\right) - \frac{1}{4}(2x) = x \ln x + \frac{1}{2}x - \frac{1}{2}x = x \ln x$$

$\blacksquare$