some basic integration-by-parts problems

1.

$$\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{x} \cdot \frac{1}{3}x^3 \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2 \, dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$$

$$\begin{array}{lll} u = \ln x & \text{and} & dv = x^2 \, dx \\ du = \frac{1}{x} \, dx & \text{and} & v = \frac{1}{3}x^3 \end{array}$$

2.

$$\int \ln x \, dx = x \ln x - \int \frac{1}{x} \cdot x \, dx = x \ln x - \int 1 \, dx = x \ln x - x + C$$

$$\begin{array}{lll} u = \ln x & \text{and} & dv = dx \\ du = \frac{1}{x} \, dx & \text{and} & v = x \end{array}$$

3.

$$\int \tan^{-1} x \, dx = x \tan^{-1} x - \int \frac{x}{1+x^2} \, dx = x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + C$$

$$\begin{array}{lll} u = \tan^{-1} x & \text{and} & dv = dx \\ du = \frac{1}{1+x^2} \, dx & \text{and} & v = x \end{array}$$

4.

$$\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x + C$$

$$\begin{array}{lll} u = x & \text{and} & dv = e^x\,dx \\ du = dx & \text{and} & v = e^x \end{array}$$

5. After applying integration by parts, use your answer from #4.

$$\int x^2e^x\,dx = x^2e^x - \int 2xe^x\,dx = x^2e^x - 2(xe^x - e^x) + C$$

$$= x^2e^x - 2xe^x + 2e^x + C$$

$$\begin{array}{lll} u = x^2 & \text{and} & dv = e^x\,dx \\ du = 2x\,dx & \text{and} & v = e^x \end{array}$$

6.

$$\int 2x\sec^{-1}x\,dx = x^2\sec^{-1}x - \int \frac{x^2}{x\sqrt{x^2-1}}\,dx$$

$$= x^2\sec^{-1}x - \int \frac{x}{\sqrt{x^2-1}}\,dx = x^2\sec^{-1}x - \sqrt{x^2-1} + C$$

$$\begin{array}{lll} u = \sec^{-1}x & \text{and} & dv = 2x\,dx \\ du = \frac{1}{x\sqrt{x^2-1}}\,dx & \text{and} & v = x^2 \end{array}$$